Reg. No. ....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2011

(CCSS)

Mathematics—Core Course

MM 5B 05—VECTOR CALCULUS

Time : Three Hours | Maximum Weightage : 30

Answer all the **twelve** *questions.*

1. A vector that is perpendicular to both of the vectors A and B is ————
2. Find parametric equations of the line through the points P (-2, 0, 3) and Q (3, 5, —2).
3. Write a vector normal to the plane ax + by + cz = d.
4. The Cartesian equation of the surface $z = r^2$ is ——
5. A particle moves along the curve x = t2, z = t3. Find the velocity at t = 1.
6. The curvature of a straight line is ————
7. Domain of the function ——$y, z) = \frac{1}{xyz}$ is ————
8. Find dy/dx if x^2 + sin y 2y = 0.
9. Find the gradient of the function $f(x, y) = y - x$ at (2, 1).
10. Write the Taylor's formula for $f(x, y)$ at the origin.
11. State Fubini's theorem (first form) for calculating double integrals.
12. Find the divergence of F $= 2xz\, i - xy\, j = z\, k$.

(12 x $\frac{1}{4}$ = 3)

Answer all the **nine** *short answer questions.*

13. Find the angle between the planes 3x – 6y.— 2z = 15 and 2x + y – 2z = 5.
14. Find the spherical coordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.
15. Show that the function $f(x, y, z) = x^2 + y^2 - 2z^2$ satisfies the Laplace's equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{} =$.
16. Find the derivative of $xe^y + \cos(xy)$ at (2, 0) in the direction of A $= 3\, i - 4\, j$
17. Find the local extreme values of the function $f(x, y) = xy$.
18. Find the area enclosed by the lemniscate $r^2 = 4 \cos 2$
19. Evaluate $\int\int\int \frac{1}{xyz}\, dzdydx$

Turn over

20. **Find the circulation of the field** $(x \quad y) \quad + xj$ **around the** cır

$r(t) = \cos t \quad + \sin j, 0 \quad t \quad 2\pi$.

21. **Find the divergence of F** $= (x2 - y)\vec{i}\ (xy - y^2)j$.

(9 x 1 =4:

Answer any **five** *short essay questions.*

22. **Find the unit tangent vector, Principal unit normal vector, Binormal vector curvature and torsi at** t **for the curve** $r(t) = 3\sin t\,\vec{i} + 3\cos t\,j + 4t\,k$.

23. **Find the linearization of** $f(x, y) = {}^2 - xy + {}^1\ y^2 + 3$ **at the point of (3, 2).**

24. **Find the angle between the surfaces** $x^2 + y^2 + z^2 = 9$ **and** $z = x^2 + y^2 - 3$ **at the point (2, —1, 2).**

25. **Find the average value of F** $(x, y, z) = xyz$ **over the cube bounded by the coordinate planes and the planes x = 2, y = 2 and** $z = 2$ **in the first octant.**

26. **Show that F** $(ex \cos y + yz)\,i + (xz - ex \sin y)\,3 + (xy + z)\,k$ **is conservative and find a potential function for it.**

27. **Using Green's theorem evaluate the integral** $y^{-1}y \ \ y\,dx$ **where C is the square cut from the first quadrant by the lines x = 1, and y = 1.**

28. **Find a parametrization of the cone** z $\overline{}, 0 < z < 1$.

x 2 =

Answer any two essay questions.

29. **Find the points closest to the origin on the hyperbolic cylinder** $x^2 - z^2 - 1 = 0$.

30. **Evaluate** $\int_0^1 \int_0^{1-x} \sqrt{x + y}\,(y\ 2x)^2\,dydx$ **by applying the transformations** $u = x + y$ **and** $v = y - 2$

31. **State Divergence theorem. Verify Divergence theorem for the field** $F = x\,i + y\,j + z\,k$ **over the sphere** $x^2 + y^2 + z2 = a2$.

(2 x 4 = 8