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FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2011

(CCSS)

Mathematics—Core Course

MM 5B 06—ABSTRACT ALGEBRA

Time : Three Hours | Maximum Weight : 30

Questions *from 1 to* **12** *are* **compulsory.**
Each has weight ¼.

1. The smallest non abelian group has ——— elements.
2. The order of the identity element in any group G is
3. A cyclic group with only one generator can have at most __ elements.
4. Write the number of cosets of 5Z in Z.
5. The Klein 4-group has bow many proper sub groups ?
6. The total number of subgroups of Z_{12} is
7. State true or false. "A subgroup of a group is a left coset of itself'.
8. The field Z_5 has how many zero divisors
9. How many unit elements are there in the ring
10. 'The alternating group A_5 has how many elements ?
11. State true or false : Z is a sub field of Q.
12. Write the number of generators of the group Z under addition.

($12 \times {}^1/_4 = 3$)

Short Answer Type Questions

Answer all questions.

13. Let G be a group and suppose that a* $b* c = e$ V a, b, c E G) . Show that $b* c* a = e$.
14. If G is an abelian group with identity e, then-all elements x of G satisfying $x^2 = e$ form a sub group of G.
15. Prove that every. cyclic group is abelian.
16. Prove that ever permutation a of a finite set is a product of disjoint cycles.
17. Exhibit the left and right of the sub group 4Z of Z.
18. Let ϕ be a homomorphism of a group G into a group G'. If $a \in G$, then prove that *(I) (a))'*
19. Find the value of the product (11) * (– 4) in Z_{15}.
20. Prove that every field F is an Integral Domain.
21. Is Q over R a vector space ? Verify.

($9 \times 1 = 9$)

Turn over

Short Essay Questions

Answer any **five** *questions.*

22. Let * be defined on Q by $a * b = \frac{ab}{2}$ Show that (Q^+, *) is an abelian group.

23. Show that intersection of sub groups H_i of a group G for $i \in I$ is again a sub group of G. Wha[t] about union of two sub groups ?

24. Describe the symmetric group S_3.

25. Prove that every prime order group is cyclic.

26. Show that cancellation law holds in a ring R if and only if R has no zero divisors.

27. Define a vector space. Give an example.

28. Show that {1, x, x^2} form a basis for P_2 (x), the collection of all polynomials of degree at most 2.

(5 x 2 1

Essay Question.

Answer any two questions.

29. (a) Define an abelian group.

(b) Describe Klein 4 - group V.

(c) Write all proper sub groups of V. Specific which are abelian

30. (a) Define the term orbit, cycle and transposition with respect to a permutation.

(b) Prove that any permutation of a finite set with at least two elements is a product [o]f transpositions.

(c) Define even and odd permutation. Prove that the product (1, 4, 5, 6). (2, 1, 5) is an odd permutation.

31. (a) State and prove Lagrange's theorem.

(b) Prove that the order of an element of a finite group divides the order of the group.

(c) Find the order of the element 2 in the group (Z_5, +).

(2 x 4 = 8)