....

Reg. No......................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2013

(U.G.–CCSS)

Mathematics—Core Course

MM 5B 06—ABSTRACT ALGEBRA

Time : Three Hours | Maximum : 30 Weightage

Answer all *twelve* questions.

1. Define an abelian group.
2. Order of a finite group G is ——————
3. In the addition group of integers the order of every element zero is ——————
4. Give an example of a cyclic group.
5. State True *or* False :

   If $a \in H = H$ then Ha = aH = H, where H is a subgroup of a group.
6. S. has ——————elements.
7. State True *or* False

   Every permutation is a one-to-one function.
8. Define a transposition.
9. State True *or* False :

   The set 9 of rational numbers is not a ring w.r.t. ordinary addition and multiplication.
10. Write the smallest subspace of any vector space.
11. Is it true that if the set $S = \{u_1, u_2, \quad u_k\}$ of a vector space V is L.D. then every superset of S is also L.D. ?
12. Define the basis of a vector space.

(12 x $\frac{1}{4}$ = 3 weightage)

Short answer questions.

Answer *all* questions.

13. What is the dimension of a vector space.
14. Define the span of a set.
15. Give an example of a linearly independent set.

Turn over

16. Define a homomorphism.

17. Give an example of a field.

18. What is the order of $\mu = (1, 4)\ (3, 5, 7, 8)$.

19. Consider $a - \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}, \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$, compute $a\,\beta^{\hat{}}$.

20. Show that $\{1, —1, i, –$ is a cyclic group.

21. Examine whether G = {-3, – 2, —1, 0,1, 2, 3} for the operation +, is a group.

(9 x 1 = 9 weightage)

Answer any *five* questions.

22. Prove that the set $G = \{a + b\sqrt{\,}\ : a, b, E\ R\}$ forms a group under multiplication.

23. Prove that if a, b are any two elements of a group G then $(a \bullet b)^2 = a^{\hat{}} b^{\hat{}}$ iff G is abelian.

24. Prove that every group of prime order is cyclic.

25. Let V = R_3 be the vector space. Let $U = \{u = (x_1, x_2, X3\ E\ V\ I\ x1 + X2 + X3 = 0\}$. Show that U is subspace of V.

26. Check whether the set :

$S = \{(1,1,0), (1,0,1), (0,1,1)\}$.

linearly independent in V_3.

27. If p is prime then prove that Z_p is a field.

28. Prove that if a finite group of order n contains an element of order n then the group must be cyclic.

(5 x 2 = 10 weightage)

Answer any *two* questions.

29. Show that a non-empty subset H of a group G is a subgroup of G iff $ab^{\hat{}}\ E$ H for all a, b E H.

30. Prove that every finite integral domain is a field.

31. Suppose $S = \{V1, V2, \ldots, V_k$ is an ordered set of a vector space V. If $V_1 \neq 0$ then prove that the set S is L.D. iff one of the vectors of $\{V_1, V_2, \ldots, V_k\}$ belongs to the span of remaining other vectors of the set S.

(2 x 4 = 8 weightage)