Reg. No. ....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2013

(UG-CCSS)

Mathematics (Core Course)

MM 5B 05—VECTOR CALCULUS

Time : Three Hours Maximum : 30 weightage

I. Answer all the *twelve* questions :

1 Find the parametric equation of the line through the points P (-3, 2, -3) and Q (1, -1, 4).

2 Find the angle between the vectors $\mathbf{A} = i - 2\mathbf{j} - 2\mathbf{k}$, $\mathbf{B} = 6i + 3\mathbf{j} + 2k$.

3 Find a vector perpendicular to both $A = 2i + j + k$ and $\bar{B} = -4\mathbf{i} + 3j + k$

4 Find the equation of the plane through P_0 (-3, 0, 7) and perpendicular to $n = 5i + 2\mathbf{j} - k$.

5 The equation $x = y^2 - z^2$ represents the surface of a __________

(a) Ellipsoid. (b) Cylinder.

(c) Cone. (d) Hyperbolic paraboloid.

6 Find the equation of the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical co-ordinates.

7 Find the unit tangent vector to the helix $\mathbf{r}\,(t) = \cos t\, i + \sin t\, j + t\mathbf{k}$.

8 Find the domain and range of the function $w = \overline{x^2 + y^2 + z^2}$

9 Find $\lim\limits_{\substack{(x,\, y) \to (1,\, 1) \\ x \neq 1}} \left\| \dfrac{xy - y - 2x + 2}{x - 1} \right|$

10 If $w = x^2 + y^2 - z + \sin t$ and $x + y = t$, find $\left(\dfrac{\partial w}{\partial y}\right)$ and

Turn over

11 Find the gradient of $g(x, y, z) = e^z$ in $(x^2 + y^2)$

12 State the Fubini's theorem (first form).

(12 x ¼ = 3 weightage)

II. Answer *all* the nine questions :

13 Find the point where line $= \frac{8}{3} + {}^2t$, $y = -2t$, $z = 1 + t$ intersects the plane $3x + 2y + 6z = 6$.

14 Find the spherical and cylindrical equation of the hemisphere $x^2 + y^2 + (z-1)^2 = 1$, $z < 1$.

15 Show that $u(t) = \sin t\, i + \cos t\, j + \sqrt{3}\, k$ has a constant length and is orthogonal to its derivative.

16 Show that the function $f(x, y, z) = 2z^3 - 3(x^2 + y^2)z$ satisfies the Laplace's equation.

17 Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point (2, 0) in the direction of $A = 3i - 4j$.

18 Find the saddle point if any of the function $f(x, y) = x^2 + xy + 3x + 2y + 5$.

19 Calculate $\iint_R \frac{\sin x}{x}\, dA$ where R is the triangle in the xy plane bounded by the x-axis, the line $y = x$ and the line $x = 1$.

20 Find the work done by $F = xyi + yj - yzk$ over the curve $\bar{r}(t) = ti + t^{\hat{}} j + tk$, $0 < t \quad 1$.

21 Evaluate $\int_0^1 \int_0^{1-z} \int_0^2 dx\, dy\, dz$.

(9 x 1 = 9 weightage)

III. Answer any *five* questions from seven :

22 Find the unit tangent vector, normal vector and binormal for the curve

$F(t) = (\cos t + t \sin t) \quad + (\sin t - t \cos t) j + 3k$.

23 Find the linearization of $f(x, y) = x^2 - xy + \frac{}{2} + 3$ at the point (3, 2)

24 Find the derivative of $f(x, y, z) = in\ (2x + 3y + 6z)$ at $p(-1, -1, 1)$ in the direction of

A= 2i+ 3j+ 6k.

25 Find the average value of $F(x, y, z) = x^2 + y^2 + z^2$ over the cube in the first octant bounded by the co-ordinate planes and the planes $x = 1$, $y = 1$ and $z = 1$.

26 Show that $F = (y + z)i + (x + z)\, j + (x + y)\, k$ forms a conservative force field and find its potential function.

27 Apply Green's theorem to evaluate $(y^2\, dx + x^2\, dy)$ where C is the triangle bounded

by $x = 0$, $x + y\ \ 1$, $y = 0$.

28 Integrate $g(x, y, z) = xyz$ over the surface of the cube cut-off by the first octant by $x = 1$, $y = 1$, $z = 1$.

(5 x 2 = 10 **weightage**)

IV. Answer any *two* questions :

29 Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.

30 Use Taylor's theorem for $f(x, y)$ to find a quadratic and cubic approximation off $(x, y) = x\, e^y$ at origin.

31 Use Stoke's theorem to evaluate $F.d\bar{r}$ if $F = xzi + xyj + 3x\, zk$ where C is the boundary of

the portion of the plane $2x + y + z = 2$ is the first octant traversed in counterclockwise sense.

(2 x 4 = 8 **weightage**)