

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2013

(UG–CCSS)

Mathematics (Core Course)

MM 5B 08—DIFFERENTIAL EQUATIONS

Time Three Hours

Maximum 30 Weightage

Part A*Answer all twelve questions.*

1. State $f(x) = x \cos x$ is even or odd.
2. Solve $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$.
3. Write the order of the differential equation $\frac{y}{dx} + 2 \left(\frac{dy}{dx} \right)^2 + \frac{dy}{dx} + y = 0$.
4. Write the necessary condition for the differential equation $M(x, y) dx + N(x, y) dy = 0$ to be exact.
5. Show that $(Ax, By) dx + (Cx, Dy) dy = 0$ is exact if $B = C$.
6. Verify that $\sin x$ is a solution of $\frac{d^2 y}{dx^2} + y = 0$.
7. Write the homogeneous equation of $\frac{dy}{dx} = \frac{y}{x}$.
8. Laplace transform of t is _____
9. If $L\{F(t)\} = f(s)$, then $L\{e^{-st} F(t)\} =$ _____
10. Find $(F * G)(t)$ if $F(t) = 1$, $G(t) = 1$.
11. $L\{e^{at} \sin bt\} =$ _____
12. Show that $(x^2 + y^2) dx + (y^2 + x^2) dy = 0$ is exact.

(12 x 3 = 36 weightage)

Turn over

Answer all questions.

13. Solve $\frac{dy}{dx} \cdot \frac{-y^2}{1-x^2} = 0$
14. Define a homogeneous differential equation.
15. Find the integrating factor of $(1+xy) ydx + (1-xy) xdy = 0$.
16. Determine $N(x, y)$ such that the equation $(x^3 + xy) dx + N(x, y) dy = 0$ is exact.
17. Find the Laplace transform of $\cos at$.
18. Find $(F * G)(t)$ if $F(t) = t$, $G(t) = e^t$.
19. Determine whether $\sin 7x$ is periodic. If so find its fundamental period.
20. Find the Laplace transform of $2e^{-3x} \cos 2t$.
21. Find the Wronskian of $\sin x$ and $\cos x$.

(9 x 1 = 9 weightage)

Answer any five questions from seven.

22. Solve $x^2 y^2 \frac{dy}{dx} = xy$.
23. Solve the initial value problem $\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 25y = 0$, $y(0) = -3$, $y'(0) = -1$.
24. Transform the equation $u'' + 2u' + 2u = 0$ into a system of first order equation.
25. If $\{F(t)\} = f(s)$ then prove that $L\{e^{at} F(t)\} = f(s-a)$.
26. Find the inverse transform of $\frac{3s+7}{s^2-2s-3}$.
27. Using Convolution property, find $L\left\{\frac{1}{s(s^2+a^2)}\right\}$.
28. Solve the boundary value problem $y'' + 2y = 0$, $y(0) = 1$, $y(\pi) = 0$.

(5 x 2 = 10 weightage)

Answer any two questions.

29. Find the integrating factor and hence solve

$$x^2 y \, dx - (x^3 + y^2) \, dy = 0.$$

30. Solve by the method of undetermined coefficients

$$\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 3y = 2e^{-x}.$$

31. Solve by the method of variation of parameters

$$\frac{d^2 y}{dx^2} + y = \tan x.$$

(2 x 4 = 8 weightage)