

**FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2014**

(UG—CCSS)

Core Course—Mathematics

MM 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours

Maximum : 30 Weights

**Part A**

Answer **all** questions.

1. Give an example of a function.
2. State Cantor's theorem.
3. What is the absolute value of -10 ?
4. State the completeness property of  $\mathbb{R}$ .
5. Give Euler number as the limit of a sequence.
6. Show that  $\{x_n\}$  is a Cauchy sequence.
7. State Cauchy convergence criterion.
8. If  $c > 1$ , find  $\lim_{n \rightarrow \infty} c^n$
9. Show that  $[0, 1]$  is not open.
10. If a set is not open will it imply that the set is closed ?
11. State de Moivre's formula.
12. Find  $\arg z$

(12 x  $\frac{1}{4}$  = 3 weights)

**Part B**

Answer **all** questions.

13. For any three sets A, B and C prove that  $A - (B \cup C) = (A - B) \cap (A - C)$ .
14. Define sequence. Give an example of a sequence.
15. Find  $\lim_{n \rightarrow \infty} \frac{2}{n+1}$
16. Show that the sequence  $(0, 2, 0, 2, \dots, 0, 2, \dots)$  does not converge to 0.
17. Find  $\lim_{n \rightarrow \infty} b^n, 0 < b < 1$ .
18. Define Cantor set.

19. Show that  $\operatorname{Re}(iz) = -\operatorname{Im}(z)$ .
20. Show that  $e^{i\theta} = \cos \theta + i \sin \theta$ .
21. Prove that  $\sin 2\theta = 2 \sin \theta \cos \theta$ . —————

(9 x 1 = 9 weightage)

### Part C

Answer any **five** questions.

22. Prove that the set  $\mathbb{Q}$  of rational numbers is denumerable.
23. Find infimum and supremum of  $\{ \frac{1}{n} : n \in \mathbb{N} \}$ .
24. Prove that the set of real numbers is not countable.
25. Show that the intersection of any finite collection of open sets in  $\mathbb{R}$  is open.
26. Show that  $2^{5t} = 1 \pm 2i$ .
27. Show that  $(\sqrt{3} + i)^{64} = 64(\sqrt{3} + i)$ .
28. If  $(x_n)$  is a convergent sequence and if  $x_n \neq L$ ,  $n \in \mathbb{N}$  then show that  $\lim_{n \rightarrow \infty} x_n = L$ .

(5 x 2 = 10 weightage)

### Part D

Answer any **two** questions.

29. Find the rational number equivalent to  $\frac{1}{\sqrt{2}}$ .
30. Prove that (i)  $\lim_{n \rightarrow \infty} \frac{2n+1}{n^2} = 0$  (ii)  $\lim_{n \rightarrow \infty} (\sin n/n) = 0$ .
31. Find all the values of  $(-8i)^{1/3}$ .

(2 x 4 = 8 weightage)