C 31126 (Pages : 3) Name..................................

Reg. No..............................

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS—UG)

Mathematics

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*

1. The product rule for natural logarithm is ————————.
2. $\lim_{x\to 0} \frac{3x - \sin x}{x} =$ ————————.
3. The Hyperbolic cosecant is defined as ————————.
4. Let $\{a_n\}$ be a sequence of real numbers. If $a_n \to L$ and if f is a function that is continuous at L and defined at all a_n, then ————————.
5. The series $\sum_{n=1}^{\infty} n^2$ diverges because ————————.
6. Suppose that $a_n > 0$ and $b_n > 0$ for all $\geq N$. If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges then ————————.
7. The first two terms in the Maclaurin series expansion of $f(x) = xe^x$ is ————————.
8. The first two terms in the expansion of $f(x) = \frac{1}{3} x \cos x$ is ————————.
9. The remainder of order n of $R_n(x)$ in Taylor's Formula is ————————.
10. The eccentricity of the conic section $r = \frac{6}{2 + \cos\theta}$ is ————————.
11. The standard form of Hyperbola if $e = 3$ and vertices $(0, \pm 1)$ is ————————.
12. The foci of ellipse . $9x^2 + 10y^2 = 90$ is ————————.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. Define Hyperbolic function and Exponential function.

14. Define natural logarithm. Give examples.

15. Find $\lim_{x \to 0} + \sqrt{x}$ in x.

16. Let $\sum a_n \sum c_n$ and $\sum d_n$ be series with non negative terms and suppose that for some integer $N, d_n \le a_n \le c_n, \forall n \ge N$. Then write the conditions for which the series $\sum a_n$ converges and diverges ?

17. Determine whether the series $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$ converges or diverges ?

18. Determine whether the Alternating series $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$ converges or diverges ?

19. Define Power series representation of a function about the point $x = a$.

20. Find the power series representation of $f(x) = \sin x$ about $x = 0$.

21. Define the radius of convergence of a power series.

22. Define eccentricity e of a conic section. Give examples.

23. Write the polar equation of an ellipse.

24. Sketch the circle $r = 6 \sin \theta$.

(9 × 2 = 18 marks)

Part C (Short Answer Type)

Answer any **six** *questions.*

25. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$. converge or diverge?

26. Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n + 5}{3^n}$.

27. Determine whether the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n\cdot 3^n}$ converge or diverge ?

28. Expand $f(x) = x^4 + x^2 + 1$ as Taylor series about a point $a = -2$.

29. Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} x^n$.

30. Discuss about the convergence of Taylor series. Give examples.

31. Find the eccentricity and directrix of the parabola $r = \frac{25}{10 - 5\cos\theta}$. Also sketch the conic.

32. Identify the conic section and hence find the centre, vertex, foci, asymptotes of $x^2 + y^2 - 2x - 2y = 0$.

33. Find the polar equation of : (i) $r\sin\theta = 2, e = 1/2$; (ii) $r\sin\theta = -6, e = 1/3$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*

34. Determine whether the series

(i) $\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n^2}\right)$ converge ?

(ii) Does the series $\sum_{n=1}^{\infty}\frac{(n+1)(n+2)}{n!}$ converge ?

35. Find the values of x for which the replacement for $\sin x$ with an error of magnitude no greater than 3×10^{-4} is possible where $\sin x = x - \frac{x^3}{3!} + \ldots\ldots$

36. Describe about polar co-ordinates and polar equation of a conic. Sketch the region defined by the polar co-ordinate inequalities

(i) $0 \le r \le 6\cos\theta$.

(ii) $-4\sin\theta \le r \le 0$.

(2 × 10 = 20 marks)