C 23294 (Pages : 4) Name..............................

Reg. No..............................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2017

(CUCBCSS—UG)

Mathematics

MAT 4B 04—THEORY OF EQUATIONS, MATRICES AND VECTOR CALCULUS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions.*

1. If $2+\sqrt{3}$ is a root of $x^4-2x^3-22x^2+62x-15=0$, without solving the equation completely, state the other root.

2. If $\alpha, \beta, \gamma \ldots$ are the roots of $f(x) = 0$, then what is the equation whose roots are $-\alpha, -\beta, -\gamma \ldots$?

3. If α and β are the roots of $lx^2+mx+n=0$ find $\alpha^2+\beta^2$.

4. Remove the second term from the equation $x^3-6x^2+4x-7=0$.

5. What is the rank of a non-singular matrix of order n ?

6. Find the row reduced Echelon form of the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \end{bmatrix}$.

7. Find the characteristic root of $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

8. State Cayley-Hamilton theorem.

9. Fill in the blanks :

   The characteristic roots of a diagonal matrix are the same as its ————————.

10. Find a vector parallel to the line of intersection of the planes $3x - 6y - 2z = 15$, $2x + y - 2z = 5$.

11. Find a Cartesian equation for the surface $z = r^2$ and identify the surface.

12. Find the unit tangent vector of :

    $r(t) = (2\cos t)i + (2\sin t)j + \sqrt{5}\, t\, k$.

(12 × 1 = 12 marks)

Turn over

Section B

Answer **all** *questions.*

13. If α, β, γ are the roots of $x^3 - x - 1 = 0$, find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$.

14. Solve $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$, given that the roots are in A.P.

15. Form an equation whose roots are increased by 2 of the equation $2x^3 + 3x^2 - x - 1 = 0$.

16. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$.

17. Compute the inverse of $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$.

18. State Sylvestee's Law of Nullity.

19. Show the characteristic roots of a Hermitian matrix are all real.

20. Find an equation for the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical co-ordinates.

21. Find the principal unit normal vector N for a curve $r(k) = (2t+3)i + \left(5-t^2\right)j$.

(9 × 2 = 18 marks)

Section C

Answer any **six** *questions.*

22. Find the rational roots of the equation $2x^3 - 3x^2 - 11x + 6 = 0$.

23. Solve the equation $x^3 - 7x^2 + 36 = 0$, given that the difference between two of its roots is 5.

24. Solve the reciprocal equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

25. Reduce to the normal form and find the rank of $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$.

26. Obtain the row reduced echelon form to find the rank of $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$.

27. Solve the system of equations :

$$x + 3y - 2z = 0$$
$$2x - y + 4z = 0$$
$$x - 11y + 14z = 0.$$

28. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify Cayley-Hamilton theorem.

29. The vector $r(t) = (2\cos t)i + (3\sin t)j + 4t\,k$ gives the position of a moving body at time t. Find the body's speed and acceleration. When $t = \pi/2$ find speed.

30. Find curvature for the helix :

$r(t) = (a\cos t)i + (a\sin t)j + btk\,;\, a,b \geq 0,\, a^2 + b^2 \neq 0.$

(6 × 5 = 30 marks)

Section D

Answer any **two** *questions.*

31. (a) Discuss the nature of roots of the equation :

$x^9 + 5x^8 - x^3 + 7x + 2 = 0$ using Descarte's rule of signs.

(b) Solve $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$, which is a reciprocal equation of second type.

32. (a) Find characteristic vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ corresponding to any one characteristic root.

(b) Obtain the inverse of the above matrix using Cayley-Hamilton theorem.

33. (a) Find the length of the indicated portion of the curve :

$$r(t) = t\,i + \frac{2}{3} t^{3/2} k \,;\, 0 \le t \le 8.$$

(b) Show that the curvature of a circle of radius a is $\frac{1}{a}$.

(2 × 10 = 20 marks)