C 30307 (Pages : 3) Name..................................

Reg. No..............................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS—UG)

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*

Each question carries 1 *mark.*

1. Define a denumerable set.
2. State trichotomy law of real numbers ?
3. State triangle inequality for real numbers.
4. Find all x satisfying $|x-3|=|x-5|$.
5. State the Archimedian property of the set of natural numbers.
6. Write the condition for a nested sequence of real numbers to have a unique common point.
7. State the ration test for the convergence of a real sequence.
8. Explicitly state monotone subsequence theorem.
9. Write the statement of Bolzano-Weierstrass theorem for a sequence.
10. Define Cauchy sequence.
11. Find Arg (z) if $z=\frac{i}{-2-2i}$.
12. Express $\left(\sqrt{3}-i\right)^7$ in the exponential form.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **ten** *out of* **fourteen** *questions.*
Each question carries 4 *marks.*

13. Prove that the collection of all finite subsets of $\mathbb{N}$ is countable.

14. If $x > 1$, prove that $(1+x)^n \geq 1+nx$ for all $n \in \mathbb{N}$.

15. If $a \geq 0$ and $b \geq 0$, prove that $a < b$ if and only if $\sqrt{a} < \sqrt{b}$.

16. Sate and prove density theorem.

17. If $y > 0$ prove that there is an n_y in $\mathbb{N}$ such that $n_y - 1 \leq y < n_y$.

18. Define the supremum and the infimum of a set S. Find them for the set $S = \left\{1 - \frac{(-1)^n}{n}, n \in \mathbb{N}\right\}$.

19. Prove that there is no rational number whose square is two.

20. Prove that there is at most one limit for the sequence of real numbers.

21. For $0 < b < 1$ show that $\lim b^n = 0$.

22. Prove that every Cauchy sequence of real numbers is bounded.

23. Discuss the convergence of the sequence $x_n = \left(1 + \frac{1}{n}\right)^n$, n in $\mathbb{N}$.

24. Prove the triangle inequality for the complex numbers algebraically.

25. Find all values of $(-8i)^{\frac{1}{3}}$.

26. Find the rectangular form of $\left(\sqrt{3} - i\right)^6$ and the principal value of the amplitude.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of* **nine** *questions.*
Each question carries 7 marks.

27. Show that the set of real numbers is uncountable.

28. Prove that there is no rational number x whose square is 3.

29. Sate and prove Cantor's theorem.

30. State and prove the characterization theorem for intervals.

31. Define contractive sequence and show that every contractive sequence is a Cauchy sequence.

32. Establish that every monotone sequence is convergent if and only if it is bounded.

33. Discuss the convergence of the following (x_n) where :

(i) $x_n = \left(1 + \frac{1}{n^2}\right)^{2n^2}$, (ii) $x_n = \frac{\sin n}{n}$.

34. Show that a real sequence is convergent if and only if it is Cauchy.

35. Find the square roots of $-\sqrt{3i} + 1$ and express them in rectangular form.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of* **three** *questions.*
Each question carries 13 marks.

36. (a) Show the existence of a positive real number in detail whose square is 2.

(b) Show that between any two real numbers there is an irrational number.

37. (a) Prove that every bounded sequence of real numbers has a converging subsequence.

(b) Show that a monotone sequence of real numbers is properly divergence if and only if it is unbounded.

38. (a) If $\lim (x_n) = x$ and $\lim (y_n) = n$ be sequences of non-zero reals that converge to x and $y \neq 0$ respectively. Prove that $\lim \left(\frac{x_n}{y_n}\right) = \frac{x}{y}$.

(b) Discuss the convergence of (i) $x_n = \frac{(-1)^n n}{n^2 + 1}$ and (ii) $y_n = \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ where $0 < a < b$.

(2 × 13 = 26 marks)