C 21563 (Pages : 2) Name..................................

Reg. No..................................

SIXTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, MARCH 2017

(UG–CCSS)

Mathematics

MM 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 30 Weightage

Section A

Answer **all** *questions.*
Each question carries ¼ weightage.

1. The real part of an analytic function is harmonic. Say True *or* False.

2. What is the value $\lim_{z \to \infty} \frac{2z^3 - 1}{z^2 + 1}$?

3. What is the imaginary part of z^3 ?

4. $e^{2+3\pi i}$ is ———.

   (a) e^2 ; (b) e ; (c) 1 ; (d) $-e^2$.

5. Express the sin hx in terms of exponential function.

6. Give an example of a complex function which is entire and bounded.

7. What is the parametric form of the circle $|z| = 4$?

8. Find pole of the function $f(z) = \frac{z-1}{z^2-1}$.

9. Determine the singular points of $\frac{z+1}{z^2 - 2x}$.

10. Write the formula for residue of $f(z)$ at a pole $z = a$ of order m.

11. For an analytic function $f(z)$ if $v_x = -2y$ *and* $v_y = 2x$ then find $f^1(z)$.

12. Find the value of $\oint_{|z|=1} \frac{dz}{z+2}$.

(12 × ¼ = 3 weightage)

Section B

Answer **all** *nine questions.*
Each question carries 1 *weightage.*

13. Show that the function $f(z) = e^z$ is differentiable everywhere.

14. Show that $u = e^x$ cosy is harmonic.

Turn over

15. Prove that an analytic function whose imaginary part is constant is itself a constant.

16. Test whether $f(z)=\frac{1}{z}$ is analytic or not.

17. Show that $\cosh(z+\pi i)=\cosh z$.

18. State Taylor's theorem.

19. Evaluate $\int_C \frac{\sin z}{z-\pi/2}\,dz$ where C is the unit circle.

20. Find the residue of $f(z)=\frac{1}{2z+3}$ at $z=-\frac{3}{2}$.

21. Find the singularities of the function $ze^{1/z}$.

(9 × 1 = 9 weightage)

Section C

Answer any **five** *questions.*
Each question carries 2 *weightage.*

22. Show that $u(x, y)=2x-x^3+3xy^2$ is a harmonic function and find a harmonic conjugate $v(x, y)$ of u.

23. Prove that the families of level curves $u(x, y)=c_1$ and $v(x, y)=c_2$ are orthogonal for an analytic function $f(z)=u(x, y)+iv(x, y)$.

24. Prove that $f(z) = z\text{Im}z$ is differentiable only $z = 0$ and find $f^1(0)$.

25. Using Cauchy's residue theorem evaluate $\int \frac{e^{2z}}{(z+1)^3}\,dz$ around the circle $|z|=\frac{3}{2}$.

26. Find all roots of the equation $\sinh z = i$.

27. Expand *sinz* into a Taylor's series about the pont $z=\frac{\pi}{4}$ and determine the region of convergence.

28. Evaluate $\int_C \frac{3z-4}{z(z-1)}\,dz$ where C is the circle $|z| = 2$ using Cauchy's integral formula.

(5 × 2 = 10 weightage)

Section D

Answer any **two** *questions.*
Each question carries 4 *weightage.*

29. Expand $f(z)=\frac{z}{(z-1)(2-z)}$ in a Laurent's series valie for (i) $|z| < 1$; (ii) $0 < |z-2| < 1$; (iii) $|z-1| > 1$.

30. Using contour integration find the value of $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$.

31. Evaluate using (i) Cauchy's integral formula ; (ii) residue theorem $\int_C \frac{z+1}{z^2+2z+4}$ where C is the circle $|z+1+i| = 2$.

(2 × 4 = 8 weightage)