C 21075 (Pages : 4) Name..............................

Reg. No..............................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2017

(CUCBCSS—UG)

Mathematics

MAT 6B 14 (E 02)—LINEAR PROGRAMMING

Time : Three Hours Maximum : 80 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Define convex hull of a set.

2. Examine whether the set $S = \{(x_1, x_2) : 5x_1 + 2x_2 \geq 10, 2x_1 + 5x_2 \geq 10\}$ is convex.

3. State graphical solution algorithm for an LPP involving two variables.

4. Define slack and surplus variables.

5. Reduce the following LPP to its standard form :

   Maximize $Z = x_1 - 3x_2$

   subject to the constraints :

   $-x_1 + 2x_2 \leq 15$

   $x_1 + 3x_2 = 10$

   x_1 and x_2 unrestricted in sign.

6. When does the simplex method indicate that the LPP has unbounded solution ?

7. Write the dual of the following LPP :

   Maximize $Z = 3x_1 - x_2 + x_3$

   subject to the constraints : $4x_1 - x_2 \leq 8$

   $8x_1 + x_2 + 3x_3 \geq 12$

   $5x_1 - 6x_3 \leq 13$

   $x_1, x_2, x_3 \geq 0.$

8. State Minimax theorem.

Turn over

9. What is transportation problem ?
10. State the necessary condition for the existence of feasible solution to the transportation problem.
11. Give the mathematical formulation of the assignment problem.
12. What is an unbalanced transportation problem ?

($12 \times 1 = 12$ marks)

Section B

Answer any **nine** *out of twelve questions.*
Each question carries 2 *marks.*

13. Formulate the following problem as a Linear Programming Problem : A person requires 10, 12 and 12 units of chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 units of A, B and C respectively per jar. A dry product contains 1, 2 and 4 units of A, B and C respectively per carton. If the liquid product is sold for Rs. 3 per jar and the dry product is sold for Rs. 2 per carton, how many units of each product should be purchased, in order to minimize the cost and meet requirements.
14. Prove that a hyperplane in R^n is a convex set.
15. Obtain graphically the maximum value of $z = \{\min(3x_1 - 10), \min(-5x_1 + 5)\}$ such that $0 \le x_1 \le 5$.
16. Write the characteristics of standard form of Linear Programming Problem.
17. The column vector (1, 1, 1) is a feasible solution to the system of equations : $x_1 + x_2 + 2x_3 = 4$ and $2x_1 - x_2 + x_3 = 2$. Reduce the given feasible solution to a basic feasible solution.
18. Verify Minimax theorem for the function $f(x) = \{9, 7, 5, 3, 1\}$.
19. State the general rules for converting any primal LPP into its dual.
20. Write all the steps for Vogel's Approximation method of solving a transportation problem.
21. Prove that every loop in a transportation table has an even number of cells.
22. How to solve the degeneracy in transportation problems ?
23. Write steps for solving assignment problem by Hungarian method.
24. State the difference between transportation problem and assignment problem.

($9 \times 2 = 18$ marks)

Section C

Answer any **six** *out of nine questions.*

Each question carries 5 *marks.*

25. Show that set of all convex combinations of a finite number of vectors $x_1, x_2, \ldots x_k$ in R^n is a convex set.

26. Use graphical method to solve the LPP :

$$\text{Maximize } Z = 6x_1 + 11x_2$$

subject to the constraints,

$$2x_1 + x_2 \leq 104$$

$$x_1 + 2x_2 \leq 76$$

$$x_1, x_2 \geq 0$$

27. Show that the following system of linear equations has a degenerate solution :—

$$2x_1 + x_2 - x_3 = 2 \text{ and } 3x_1 + 2x_2 + x_3 = 3.$$

28. Use simplex method to solve the LPP :

$$\text{Maximize } Z = 3x_1 + 2x_2$$

subject to the constraints

$$4x_1 + 3x_2 \leq 12$$

$$4x_1 + x_2 \leq 8$$

$$4x_1 - x_2 \leq 8 \text{ and } x_1, x_2 \geq 0.$$

29. Explain the Charne's Big-M method.

30. Prove that dual of the dual is primal.

31. Determine an initial basic feasible solution to the following transportation problem using the row minima method.

	D_1	D_2	D_3	*Supply*
O_1	50	30	220	1
O_2	90	45	170	4
O_3	250	200	50	4
Required	4	2	3	9

32. Prove that there always exist an optimal solution to a balanced transportation problem.

Turn over

33. The owner of a small machine shop has four machinists available to do jobs for the day. Five jobs are offered with expected profit for each machinist on each job as follows :

Jobs	*Machinists*			
	1	2	3	4
A	32	41	57	18
B	48	54	62	34
C	20	31	81	57
D	71	43	41	47
E	52	29	51	50

Find, by using assignment method, the assignment of machinists to jobs that will result in a maximum profit.

(6 × 5 = 30 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 10 *marks.*

34. Let $A \subseteq R^n$ be any set. Prove that $< A >$, the convex hull of A, is the set of all finite convex combination of vectors in A.

35. Use Simplex method to solve the LPP :

Minimize $Z = x_2 - 3x_3 + 2x_5$

subject to the constraints

$$3x_2 - x_3 + 2x_5 \leq 7$$

$$-2x_2 + 4x_3 \leq 12$$

$$-4x_2 + 3x_3 + 8x_5 \leq 10 \text{ and } x_2, x_3, x_5 \geq 0.$$

36. Obtain an optimum basic feasible solution to the following degenerate transportation problem :

	D_1	D_2	D_3	*Availability*
O_1	7	3	4	2
O_2	2	1	3	3
O_3	3	4	6	5
Demand	4	1	5	10

(2 × 10 = 20 marks)