D 40046 (Pages : 4) Name..................................

Reg. No...............................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2018

(CUCBCSS—UG)

Mathematics

MAT 6B 13 (E02)—LINEAR PROGRAMMING

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Define extreme point of a convex set.

2. A hyperplane is given by the equation $3x_1 + 2x_2 + 4x_3 + 7x_4 = 8$. Find in which half space does the point $(-6, 1, 7, 2)$ lie.

3. What do you mean by a degenerate basic solution ?

4. Define slack and surplus variable.

5. Reduce the following LPP to its standard form : Maximize $z = x_1 + 2x_2$ subject to the constraints,

$$2x_1 - 3x_2 \leq 3$$
$$4x_1 + x_2 \leq -4$$
$$x_1, x_2 \geq 0.$$

6. When does the simplex method indicate that the LPP has unbounded solution ?

7. Write the dual of the following LPP Min $4x_1 + 6x_2 + 18x_3$ subject to the constraints,

$$x_1 + 3x_3 \geq 3$$
$$x_2 + 2x_3 \geq 5$$
$$x_1, x_2, x_3 \geq 0.$$

8. When does the transportation problem is said to be unbalanced.

9. What is transportation problem ?

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10. Give mathematical formulation of the assignment problem.

11. State the necessary condition for the existence of feasible solution to transportation problem.

12. What do you mean by an unbalanced assignment problem ?

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Formulate the following problem as a LPP : A firm manufactures two type of products A and B and sells them at a profit of Rs. 2 on type A and Rs .3 on type B. Each product is processed on two machines M_1 and M_2. Type A requires 1 minute of processing time on M_1 and 2 minutes on M_2; Type B requires 1 minute on M_1 and 1 minute on M_2. The machine M_1 is available for not more than 6 hours 40 minutes, while machine M_2 is available for 10 hours during any working day.

14. Prove that a hyperplane in $\mathbb{R}^n$ is a convex set.

15. Solve graphically, Maximize $z = 2x_1 + 3x_2$ subject to the constraints

$$x_1 + x_2 \leq 1;$$
$$3x_1 + x_2 \leq 4;$$
$$x_1, x_2 \geq 0.$$

16. Write the characteristics of standard form of LPP.

17. Find all basic solutions of the system,

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5.$$

18. Prove that the intersection of 2 convex sets is also a convex set.

19. Show that the dual of the dual of a given primal is the primal itself.

20. Write all the steps for North-West corner rule.

21. How to solve the degeneracy in transportation problem.

22. Give an algorithm to solve assignment problem.

23. Write the steps for solving assignment problem by Hungarian method.

24. Write the main differences between assignment problem and transportation problem.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Use graphical method to solve Maximize $z = 5x_1 + 7x_2$ subject to the constraints

$$x_1 + x_2 \le 4$$
$$3x_1 + 8x_2 \le 24$$
$$10x_1 + 7x_2 \le 35$$
$$x_1, x_2 \ge 0.$$

26. Explain Charne's Big-M method.

27. Show that the set of all convex combinations of a finite number of vectors $x_1, x_2, \ldots\ldots x_k$ in $\mathbb{R}^n$ is a convex set.

28. Using Simplex method to solve the LPP : Maximize $2x_1 + 4x_2$ subject to the constraints

$$2x_1 + 3x_2 \le 48$$
$$x_1 + 3x_2 \le 42$$
$$x_1 + x_2 \le 21$$
$$x_1, x_2 \ge 0.$$

29. Let $x_1 = 2, x_2 = 4, x_3 = 1$ be a feasible solution to the system of equations $2x_1 - x_2 + 2x_3 = 2$, $x_1 + 4x_2 = 18$, reduce the feasible solution to a basic feasible solution.

30. Show that $S = (x_1, x_2, x_3) : 2x_1 - x_2 + x_3 \le 4$ in $\mathbb{R}^3$ is a convex set.

31. Prove that there always exist an optimal solution to a balanced transportation problem.

32. Solve :

	D_1	D_2	D_3	D_4	supply
O_1	21	16	25	13	11
O_2	17	18	14	23	13
O_3	32	27	18	41	19
Demand	6	10	12	15	43

33. Prove that there always exist an optimal solution to a balanced transportation problem.

(6 × 5 = 30 marks)

Turn over

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Use Simplex method to solve the LPP : Maximize $z = 2x_1 + x_2$ subject to the constraints

$$\begin{aligned} 4x_1 + 3x_2 &\leq 12 \\ 4x_1 + x_2 &\leq 8 \\ 4x_1 - x_2 &\leq 8 \\ x_1, x_2 &\geq 0. \end{aligned}$$

35. Let $A \subseteq \mathbb{R}^n$ be any set. Show that the convex hull of A, is the set of all finite convex combination of vectors in A.

36. Solve the transportation problem

	D_1	D_2	D_3	D_4	Availability
O_1	1	2	1	4	30
O_2	4	2	5	9	50
O_3	20	40	30	10	20
Demand	20	40	30	10	100

(2 × 10 = 20 marks)