**D 40041** (Pages : 4) Name..................................

Reg. No..............................

## SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2018

(CUCBCSS—UG)

Mathematics

MAT 6B 09—REAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

### Part A

*Answer all the* **twelve** *questions.*
*Each question carries* 1 *mark.*

1. State the Location of Roots Theorem.
2. Give an example of a continuous function on $A = (0, \infty)$ which is not a uniformly continuous function on $A = (0, \infty)$.
3. State the Preservation of Intervals Theorem.
4. State the Weierstrass Approximation Theorem.
5. Define the Norm of a Partition of a closed and bounded interval.
6. State the Boundedness Theorem on Riemann integration.
7. Give the Lebesgue Integrability criterion of a function.
8. State the Substitution Theorem for Riemann Integration.
9. For what value of $r$ the series $\sum_{n=1}^{\infty} r^n \sin nx$ uniformly convergent.
10. Fill in the blanks : $\lim \left( \frac{(x^2 + nx)}{n} \right) =$ .....
11. Fill in the blanks : $\beta(1/2, 1/2) =$.....
12. Fill in the blanks : $\int_0^{\infty} \sqrt{x}e^{-x}\, dx =$ .....

(12 × 1 = 12 marks)

Turn over

## Part B

*Answer any* **ten** *questions.*
*Each question carries 4 marks.*

13. State the Boundedness Theorem. Show by an example that the boundedness theorem fails if the Interval is not bounded.

14. Let $I = [a, b]$, be a closed and bounded Interval . If $f : I \to R$ is continuous on $[a, b]$ , then prove that $f([I])$ is a closed and bounded interval.

15. Show by an example that the continuous image of an open interval need not be an open interval.

16. State the "sequential criteria" for the uniform continuity of a function. Apply this result to test the uniform continuity of $f(x) = 1/x$ on $(0, 1)$.

17. Use the location of Roots Theorem to show that the equation $x^3 - x - 1 = 0$ has a root in $(1, 2)$.

18. If $f \in R[a, b]$ and $|f(x)| \leq M, \forall x \in [a, b]$, then prove that $\left|\int_a^b f\right| \leq M(b - a)$.

19. If $\phi : [a, b] \to R$ is a Step function, then prove that $\phi \in R[a, b]$.

20. If F and G are differentiable functions on $[a, b]$ and $f = F', g = G' \in R[a, b]$, then prove that $\int_a^b fG = [FG]_a^b - \int_a^b Fg$.

21. Define uniform convergence of a sequence of functions. Test uniform convergence of the sequence $(x/n)$; $x \in R, n \in N$.

22. Define uniform norm of a bounded function. Find $\|f_n - f\|_{A}$, if $(f_n)(x) = (x^n ; x \in A = [0, 1])$ and $f(x) = 0$ for $0 \leq x < 1$; $f(x) = 1$ for $x = 1$.

23. Define uniform convergence of a series of function. Show that the Uniform limit of a series of continuous functions is continuous .

24. Show that $\int_{x=a}^{b} \frac{1}{(x-a)^p} dx$ converges , if $p < 1$; and diverges if $p \geq 1$.

25. Define the Cauchy Principal value of the integral $\int_{x=-\infty}^{\infty} f(x)dx$. Find it if $f(x)=1/x$.

26. Define Beta function . Prove that $\beta(m,n)=\beta(n,m), \forall m,n>0$.

(10 × 4 = 40 marks)

## Part C

*Answer any* **six** *questions.*
*Each question carries 7 marks.*

27. State and prove the Intermidiate Value Theorem.

28. State and prove the Continuous Extension Theorem.

29. State and prove the Composition Theorem of Riemann integration.

30. Define Riemann integral of a function. If $f \in R[a,b]$, then prove that the value of the Riemann integral of $f$ is uniquely determined.

31. Prove that a sequence of bounded functions on A converges uniformly on A if and only if $\|f_n - f\|_A \to 0$.

32. Test the uniform convergence of the series $\sum_{n=1}^{\infty} \frac{\cos(nx)}{n^p}$.

33. (a) Distinguish between absolute and conditional convergence of an improper integral.

(b) Discuss the convergence of the Improper integral $\int_{x=\pi}^{\infty} \frac{\sin x}{x} dx$.

34. Evaluate $\int_a^b (x-a)^{m-1}(b-x)^{n-1} dx; m,n>1$.

35. Express the integral $\int_0^1 (x)^p (1-x^q)^n dx; p,q,n>0$ in terms of Gamma function.

(6 × 7 = 42 marks)

## Part D

*Answer any* **two** *questions.*
*Each question carries* 13 *marks.*

36. (a) Let I = $[a, b]$, be a closed and bounded Interval . If $f : \mathrm{I} \to \mathrm{R}$ is continuous on $[a, b]$ , then prove that $f$ is bounded on I = $[a, b]$ .

(b) Prove that $\beta(p, 1-p) = \int_{x=0}^{\infty} \frac{x^{p-1}}{(1+x)} dx.$

37. (a) State and prove Squeeze Theorem of Riemann Integration.

(b) Evaluate $\left(\frac{(\sin nx)}{1+nx}\right); x \geq 0.$

38. (a) State and prove the Weierstrass-M-test for uniform convergence of a Series of functions.

(b) Show that $\beta(m, n) = \int_{x=0}^{1} \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx.$

(2 × 13 = 26 marks)