C 21569 (Pages : 4) Name.....

Reg. No.....

# SIXTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, MARCH 2017

(UG—CCSS)

Mathematics

MM 6B 13(E02)—LINEAR PROGRAMMING AND GAME THEORY

(2010 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

## Part I

*Answer* **all** *questions.*
*Each question carries ¼ weightage.*

1. Define optimal solution of a Linear Programming Problem.
2. Define degenerate basic feasible solution of an LPP.
3. Define basic variables for an LPP.
4. Write the Standard form of an LPP.
5. Write the Canonical form of Maximization Problem in LPP.
6. What is meant by Integer Liner Programming.
7. In a transportation problem if travelling cost for a destination is not known, it is assumed to be ————.
8. Define a Convex set K in $\mathbb{R}^n$.
9. State True *or* False "If S be convex, then all convex combinations of elements of S lie in S".
10. Define convex hull of a set S in $E^n$.
11. In a balanced Transportation Problem the basic variables can be at most ————.
12. When will we say that a feasible solution of a Transportation Problem is basic

(12 × ¼ = 3 weightage)

## Part II

*Answer any* **nine** *questions.*
*Each question carries* 1 *weightage.*

13. Solve graphically the LPP.

Max $z = 5x_1 + 7x_2$, subject to $x_1 + x_2 \leq 4, 3x_1 + 8x_2 \leq 24, 10x_1 + 7x_2 \leq 35$, Where $x_1 \geq 0, x_2 \geq 0$.

14. Reduce the problem to Canonical form

Min $z = x_1 + x_2$, subject to $2x_1 - x_2 = 4, 3x_1 + 5x_2 = 10$ and $x_1 \geq 0, x_2 \geq 0$.

15. Verify that $A = \{(x, y) \in \mathbb{R}^2 ; x^2 + y^2 = 1\}$ is a convex set or not.

16. Prove that every hyper plane in $\mathbb{R}^n$ is convex.

17. Find the dual of :

Max $z = x_1 - x_2 + 3x_3$, subject to $x_1 + x_2 + x_3 \leq 10, 2x_1 - x_3 \leq 2, 2x_1 - 2x_2 + 3x_3 \leq 6$, Where

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$.

18. Solve algebraically

Max $z = 10x_1 + 15x_2$, subject to $x_1 + x_2 = 2$, $3x_1 + x_2 \leq 6$, and $x_1 \geq 0, x_2 \geq 0$.

19. Give a counter example to show that "union of two convex sets need not be convex".

20. Find the initial basic feasible solution for the transportation problem using NWCR.

| | Destination | | | | |
|---|---|---|---|---|---|
| Source | 21 | 16 | 25 | 13 | 11 |
| | 17 | 18 | 14 | 23 | 13 |
| | 32 | 27 | 18 | 41 | 19 |
| | 6 | 10 | 12 | 15 | |

21. Write the mathematical form of the assignment problem.

22. Use simplex method to solve

Max $z = 2x_1 + 3x_2$, subject to $x_1 + x_2 \leq 1, 3x_1 + x_2 \leq 4$, and $x_1 \geq 0, x_2 \geq 0$.

23. Solve the Transportation problem with initial b. f. s. $x_{13} = x_{21} = x_{33} = 3, x_{32} = x_{34} = 2$.

| | Destination | | | | |
|---|---|---|---|---|---|
| Source | 10 | 7 | 3 | 6 | 3 |
| | 1 | 6 | 8 | 3 | 5 |
| | 7 | 4 | 5 | 3 | 7 |
| | 3 | 2 | 6 | 4 | |

24. What is meant by a non-degenerate basic feasible solution of a TP.

(9 × 1 = 9 weightage)

## Part III

*Answer any* **five** *questions.*
*Each question carries 2 weightage.*

25. Show that the set of all feasible solutions of an LPP is a closed convex set.

26. Solve the LPP

Min $z = x_1 - 3x_2 + 2x_3$, subject to $3x_1 - x_2 + 2x_3 \leq 7, -2x_1 + 4x_2 \leq 12, -4x_1 + 3x_2 + 8x_3 \leq 10$, Where $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$.

27. Find a basic feasible solution to the following TP by Vogels Approximation Method.

| | Destination | | | | |
|---|---|---|---|---|---|
| Source | 5 | 3 | 6 | 4 | 30 |
| | 3 | 4 | 7 | 8 | 15 |
| | 9 | 6 | 5 | 8 | 15 |
| | 10 | 25 | 18 | 7 | |

28. Food X contains 6 units of Vitamin A and 7 units of vitamin B per gram and cost 12 paise per gram. Food Y contains 8 units and 12 units of A and B per gram respectively and costs 20 paise per gram. The daily requirements of vitamin A and B are at least 100 units and 120 units respectively. Formulate the above as an LPP.

29. Show that $H = \{(x, y) \in \mathbb{R}^2 ; a \leq x \leq b\}$ is a convex set.

30. Prove that the set of all feasible solutions of a system of equation $Ax = b$ is a closed convex set.

31. Use Charne's Method to solve

Max $z = -x_1 - x_2 - x_3$, subject to $x_1 - x_2 + 2x_3 = 2$, $x_1 + 2x_2 - x_3 = 1$,

Where $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$.

32. Solve the assignment problem

| | M1 | M2 | M3 | M4 | M5 |
|---|---|---|---|---|---|
| J1 | 9 | 3 | 4 | 2 | 10 |
| J2 | 12 | 10 | 8 | 11 | 9 |
| J3 | 11 | 2 | 9 | 0 | 8 |
| J4 | 8 | 0 | 10 | 3 | 7 |
| J5 | 7 | 5 | 6 | 2 | 9 |

(5 × 2 = 10 weightage)

## Part IV

*Answer* **Both** *questions.*
*Each question carries* 4 *weightage.*

33. Solve the Transportation Problem

| | Destination | | | | |
|---|---|---|---|---|---|
| Source | 6 | 1 | 9 | 3 | 70 |
| | 11 | 5 | 2 | 8 | 55 |
| | 10 | 12 | 4 | 7 | 90 |
| | 85 | 35 | 50 | 45 | |

34. Solve the Assignment problem

| | M1 | M2 | M3 | M4 | M5 |
|---|---|---|---|---|---|
| J1 | 11 | 17 | 8 | 16 | 20 |
| J2 | 9 | 7 | 12 | 6 | 15 |
| J3 | 13 | 16 | 15 | 12 | 16 |
| J4 | 21 | 24 | 17 | 28 | 26 |
| J5 | 14 | 10 | 12 | 11 | 15 |

(2 × 4 = 8 weightage)