Graphs and Combinatorics (1994) 10: 269–270

Graphs and Combinatorics

About a Conjecture on the Centers of Chordal Graphs

K.S. Parvathy, A. Remadevi, and A. Vijayakumar

Department of Mathematics and Statistics, Cochin University of Science and Technology, Cochin 682 022, India

Abstract. In this paper, a conjecture of G.J. Chang, that $d(C(G)) \leq 2$ for any connected chordal graph G with $d(G) = 2r(G) - 2$, is disproved.

1. Introduction

Let $G = (V, E)$ be a connected graph. The *distance* $d(x, y)$ from a vertex x to a vertex y is the minimum number of edges in a path from x to y. The *eccentricity* $e(x)$ of x in V is the maximum of $d(x, z)$, z in V, *radius* $r(G)$ is the minimum of $e(z)$, z in V, and *diameter* $d(G)$ is the maximum of $e(z)$, z in V.

The *center* of a graph G is defined to be $C(G) = \{x \in V | e(x) = r(G)\}$. Because of numerous applications of this concept, the structure of the center for various classes of graphs are well studied. Apart from the theorem of Jordan, that the center of a tree is either K_1 or K_2, centers of maximal outer planar graphs, 2-trees, unicyclic graphs, and median graphs are discussed in [5–7], and [3], respectively.

2. Centers of Chordal Graphs

A graph G is *chordal* (*triangulated, rigid circuit*) if it contains no cycle of length greater than three as an induced graph. Though, the center of a connected graph need not be so, it is known [2] that the center of a connected chordal graph is always connected and for such graphs $d(C(G)) < 3$, see [4]. Consequently, the results in [1], that $d(C(G)) \leq 3$ for any connected chordal graph G with $d(G) = 2r(G) - 1$ and $d(C(G)) \leq 5$ for such graphs with $d(G) = 2r(G) - 2$, are less significant.

Since $2r(G) - 2 \leq d(G)$ for any connected chordal graph, it follows that for a self-centered chordal graph G, $r(G) \leq 2$. In [4], a characterization of self-centered chordal graphs is given. These results have led to the following counter example for a conjecture in [1].

3. Counter Example

Conjecture [1]. $d(C(G)) \leq 2$ for any connected chordal graph with $d(G) = 2r(G) - 2$.

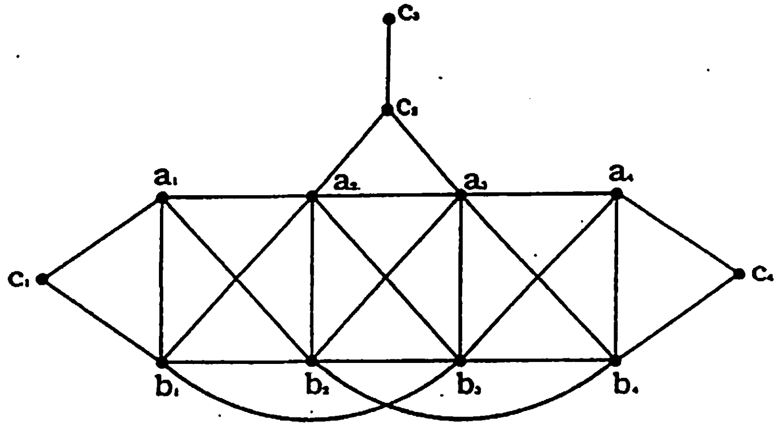

Fig. 1.

That, the conjecture is not true, can be seen from the following example.

G has $(r, d) = (3, 4)$ and $d(C(G)) = 3$. A class of such graphs can be constructed from G by the following procedure. Replace each b_i by a complete graph $K_{n_i} = \langle b_i^1, b_i^2, \ldots, b_i^{n_i} \rangle$, b_i^j is adjacent to b_j^k if b_i is adjacent to b_j also a_i or c_k is adjacent to b_i^j if they are adjacent to b_i in G.

Acknowledgement. The authors are grateful to the referee for some suggestions during the revision of the paper. The first author thanks the C.S.I.R. for awarding a research fellowship.

References

1. Chang, G.J.: Centers of chordal graphs. Graphs and Combinatorics 7, 305–313 (1991)
2. Laskar, R. and Shier, D.: On powers and centers of chordal graphs, Disc. Appld. Math. 6, 139–147 (1983)
3. Nieminen, J.: Distance center and centroid of a median graph. J. Franklin Inst. 323, 89–94 (1987)
4. Prabir Das and Rao, S.B.: Center graphs of chordal graphs. Proc. of the Seminar on Combin. and Applns., ISI, 81–94 (1982)
5. Proskurowski, A.: Centers of 2-trees. Annals of Disc. Math. 9, 1–5 (1980)
6. Proskurowski, A.: Centers of maximal outer planar graphs. J. Graph. Theory 4 (2), 75–79 (1980)
7. Truszcynski, M.: Centers and centroids of unicyclic graphs. Math. Slovaca, 35, 223–228 (1985)

Received: August 11, 1993
Revised: April 8, 1994