C 25917 **(Pages : 3)** **Name**.....

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION

APRIL 2017

(UG—CCSS)

Complementary Course

MM 2C 02—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

Answer **all** *questions.*

1. Integrate $\operatorname{sech}^2\left(x-\frac{1}{2}\right)$.
2. $\tanh^2 x = 1 -$ ———.
3. Show that $\cosh 2x = \cosh^2 x + \sinh^2 x$.
4. $\lim_{n\to\infty} \frac{\ln(n)}{n} =$ ———.
5. Give an example of a constant sequence.
6. State n^{th} Root Test.
7. Find the Taylor polynomial of order zero generated by $f(x) = \sin x$ at $x = \frac{\pi}{4}$.
8. If $f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$ Find the series for $f^1(x)$.
9. Define Absolute convergence.
10. If $f(x, y) = x \cos y + y e^x$ find $\frac{\partial^2 f}{\partial x^2}$.
11. Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$.
12. Differentiate $\tanh\sqrt{1+t^2}$.

(12 × ¼ = 3 weightage)

Turn over

Answer **all** *the questions.*

13. Evaluate $\int_0^1 \sinh^2 x \, dx$.

14. Evaluate $\int_{\frac{-\pi}{4}}^{\pi/4} \cosh(\tan\theta)\sec^2\theta \, d\theta$.

15. Investigate the convergence of $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

16. Evaluate $\int_{-\infty}^{\infty} \frac{2x\,dx}{(x^2+1)^2}$.

17. Given $a_1 = 2, a_{n+1} = (-1)^{n+1}\frac{a_n}{2}$. Write the first four terms of the sequence.

18. For what values of x do the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges absolutely.

19. Graph the set of points whose polar co-ordinates satisfy the conditions $r \leq 0$ and $\theta = \frac{\pi}{4}$.

20. Find f_x if $f(x, y) = \frac{2y}{y + \cos x}$.

21. Find an equation for the hyperbola with eccentricity $\frac{1}{2}$ and directrix $x = 2$.

(9 × 1 = 9 weightage)

Answer any **five** *questions.*

22. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \ln s$, $z = 2r$.

23. Find the derivative of $f(x, y) = x e^y + \cos(x, y)$ at the point (2, 0) in the direction of $A = 3i - 4j$.

24. Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2} y^2 + 3$ at the point (3, 2).

25. Find :

$$\lim_{(x,y)} \to (0, 0) \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}.$$

26. Find a polar equation for the circle $x^2 + (y - 3)^2 = 9$.

27. Find the Taylor series generated by $f(x) = x^4 + x^2 + 1$ at $a = 2$.

28. Investigate the convergence of the series $\sum_{n=0}^{\infty} \frac{2^n + 5}{3^n}$.

(5 × 2 = 10 weightage)

Answer any **two** *questions.*

29. Show that the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + ... + \frac{1}{n^p} + ...$ (p in a real number) converges if $p > 1$ and diverges if $p \leq 1$.

30. Find an upper bound $|E|$ of the error in the approximation $f(x, y) \approx L(x, y)$ over the rectangle $R : |x - 2| \leq 0.1, |y - 1| \leq 0.1$ for $f(x, y) = x^2 - 3xy + 5$.

31. Find the area inside the smaller loop of $r = 2 \cos \theta + 1$.

(2 × 4 = 8 weightage)