C 24763 (Pages : 4) Name.......................................

Reg. No..................................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2017

(CUCBCSS—UG)

Complementary Course

MAT 2C 02—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer **all** *questions.*

1. Find the derivative of y with respect x, where $y = \ln(\sinh x)$.

2. Evaluate $\int_5^2 \frac{dx}{1-x^2}$.

3. Find the value of $\int \frac{du}{\sqrt{a^2+u^2}}$ when $a > 0$.

4. Write the formula for the length of the curve $x = g(y), c \le y \le d$.

5. Write the limit comparison test for improper integrals.

6. Show that $\sum_{n=1}^{\infty} \frac{n+1}{n}$ diverges.

7. Find the Maclaurin series for the function e^{-x}.

8. Replace the following Cartesian equation by equivalent polar equation.

   $$xy = 2.$$

9. Find an equation for the hyperbola with $\frac{3}{2}$ eccentricity and directrix $x = 4$.

10. Evaluate $\lim_{(x,y \to (0,1))} \frac{x - xy + 3}{x^2 y + 5xy - y^3}$.

11. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$, where $f(x, y) = x^2 - xy + y^2$.

12. If $w = f(x, y, z)$, $x = g(r, s)$, $y = h(r, s)$ and $z = k(r, s)$ write $\frac{\partial w}{\partial s}$.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. Find the volume of the solid generated byrevolving the region bounded by the lines

$y = 2$, $x = 0$ and the curve $y = 2\sqrt{x}$.

14. Find the length of the curve $y = \frac{y^3}{3} + \frac{1}{4y}$ from $y = 1$ to $y = 3$.

15. Find the area of the surface generated by revolving the curve $x = \frac{y^3}{3}, 0 \le y \le 1$ about the y-axis.

16. Evaluate $\int \tanh \frac{x}{7}\, dx$.

17. Investigate the convergence of $\int_0^{\frac{\pi}{2}} \tan\theta\, d\theta$.

18. Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{4n}$.

19. Find the Maclaurin series for the function $\frac{1}{1-x}$.

20. Find the polar equation for the circle $x^2 + (y-3)^2 = 19$.

21. Find the directrix of the parabola $r = \frac{25}{10^{-5} \cos\theta}$.

22. What point satisfies the equations $r = 2, \theta = \frac{\pi}{4}$?

23. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ where $f(x, y) = \frac{1}{x+y}$.

24. State chain rule for two independent variables and three intermediate variables.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*

25. Find the volume of the solid generated by revolving the region bounded by the Curve $x = \frac{\sqrt{2}\, y}{y^2 + 1}$ and the lines $x = 0$ and $y = 1$.

26. Find the length of the curve $y = \frac{4\sqrt{2}}{3} x^{\frac{3}{2}} - 1, 0 \le x \le 1$.

27. Evaluate $\int_0^{ln 2} 4e^{-\theta} \sinh \theta \, d\theta$.

28. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1 + x^2}$.

29. Find the sum of the series $\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$.

30. (a) Graph the curve $r = 1 - \cos \theta$.

    (b) Show that the point $\left(2, \frac{\pi}{2}\right)$ lie on the curve $r = 2 \cos 2\theta$.

31. Find the points of the intersection of the curves $r^2 = 4 \cos \theta$ and $r = 1 - \cos \theta$.

32. Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2} y^2 + 3$.

33. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $= x^2 + y^2 x = r - s, y = r + s$.

(6 × 5 = 30 marks)

Turn over

Part D (Essay Type)

Answer any **two** *questions.*

34. (a) Show that if u is a differentiable function of x whose values are greaterthan 1, then :

$$\frac{d}{dx}\left(\cosh^{-1} u\right) = \frac{1}{\sqrt{u^2 - 1}} \frac{du}{dx}.$$

(b) Evaluate $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx.$

35. (a) Find all the second order partial derivatives of $f(x, y) = x^2 y + \cos y + y \sin x$.

(b) Draw the tree diagrams and chain rules for the derivatives $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$ for

$$z = f(x, y),\ x = g(t, s),\ y = h(t, s).$$

36. (a) Find a polar equation of the conic with $= \frac{1}{5}$, one focus at origin and directrix $y = -10$ corresponding to that focus.

(b) Sketch the circle $r = 2a \sin\theta$. Give polar co-ordinates for the centers and identify the radius.

(2 × 10 = 20 marks)