C 23323 (Pages : 3) Name..................................

Reg. No..................................

# FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2017

(CUCBCSS–UG)

Complementary Course

MAT 4C 04—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

## Part A

*Answer* **all** *questions.*
*Each question carries 1 mark.*

1. Find the general solution of $y^{11} + w^2 y = 0$.
2. Find $(D^2 + 3D) \cosh 3x$.
3. Write the general form of the Cauchy-Euler equation.
4. Find $L(a + bt + ct^2)$.
5. $L(f') =$ ———.

   (a) $L(f) - S f(0)$. (b) $L(f) - f(0)$.

   (c) $SL(f) - f(0)$. (d) $SL(f) - f'(0)$.
6. $L^{-1}\left(\frac{1}{s^2 + a^2}\right) =$ ———.
7. Write the second shifting theorem of Laplace transform.
8. Sketch $f(x) = |x|$ for $-\pi < x < \pi$.
9. Find $a_0$ in the Fourier series expansion of $f(x) = x^2, -\pi < x < \pi$.
10. Write the Picard's iteration formula to find a numerical solution of $y' = f(x, y), y(x_0) = y_0$.
11. Write the one-dimensional wave equation.
12. Find the period of $\cos \pi x$.

(12 × 1 = 12 marks)

## Part B

*Answer any* **nine** *questions.*
*Each question carries 2 marks.*

13. Find the general solution of $(9D^2 + 6D + 1) y = 0$.
14. Solve $x^2 y'' - 3xy' + 4y = 0$.
15. Find the Laplace transform of $(t + 1)^2 e^t$.

Turn over

16. Find $L^{-1}\left(\frac{60+6s^2+s^4}{s^7}\right)$.

17. If $f(t) = t$ and $g(t) = e^{at}$ find the convolution $(f * g)(t)$.

18. Find the Fourier cosine series of the function $f(x) = \pi - x$ in $0 < x < \pi$.

19. Prove that product of an even function and an odd function is an odd function.

20. Apply Picard's method to solve the initial value problem $y' = x^2 + y, y(0) = -1$.

21. Solve the partial differential equation $u_{xy} = u$.

22. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal rule taking $h = 1$.

23. Solve the initial value problem $y'' + y' = 0, y(0) = 5, y'(0) = -3$ using Laplace transform.

24. Derive the Euler's formula to solve the differential equation $y' = f(x, y), y(x_0) = y_0$.

(9 × 2 = 18 marks)

## Part C

*Answer any* **six** *questions.*
*Each question carries* 5 *marks.*

25. Find the general solution of $y'' + 2y' + y = 2x + x^2$.

26. Find the general solution of $(D^2 + 3D - 4)y = 8\cos 2x$.

27. Solve $y'' + y = \sec x$ by the method of variation of parameters.

28. Find the inverse Laplace transform of $\frac{1}{(s+\sqrt{2})(s-\sqrt{3})}$.

29. Solve the initial value problem by Laplace transform $y' + 3y = 10\sin t, y(0) = 0$.

30. Find the Fourier series of the function $f(x) = x^2, -\pi < x < \pi$.

31. Using Laplace transform solve the integral equation $y(t) = 1 - \int_0^t (t-\tau)\, y(\tau)d\tau$.

32. Find the deflection $u(x, t)$ of a string of length $L = 2\pi$ when $c^2 = 1$, the initial velocity is zero and initial deflection is $0.1(\pi^2 - x^2)$.

33. Evaluate $\int_1^7 \frac{dx}{x}$ using Simpson's rule by dividing [1, 7] into 6 equal parts.

(6 × 5 = 30 marks)

## Part D

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

34. Solve the initial value problem $y'' + 1.2y' + 0.36y = 4e^{-0.6x}$, $y(0) = 0$, $y'(0) = 1$.

35. Solve $y'' + 2y' + 5y = e^{-t}\sin t$, $y\,(0) = 0$. Using Laplace transform. Given $y'(0) = 1$.

36. Use improved Euler's method to determine $y$ (0.2) in two steps from $\frac{dy}{dx} = x^2 + y$, given that $y\,(0) = 1$.

(2 × 10 = 20 marks)