FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS—UG)

Complementary Course—Mathematics

MAT 1C 01—MATHEMATICS

Time : Three Hours   Maximum : 80 Marks

Part A

Answer all **twelve** *questions.*
Each question carries 1 mark.

1. Suppose $\lim\limits_{x \to c} f(x) = 5$ and $\lim\limits_{x \to c} g(x) = -2$, find $\lim\limits_{x \to c} 2f(x) g(x)$.

2. Find dy if $y = \cos 3x + x^4$.

3. Write the sum without sigma notation : $\sum\limits_{k=1}^{1} 0k^2 + 3k$.

4. Find the interval in which the function $y = x^3$ is concave up.

5. Find absolute extrema of $y = x^2$ on $(0, 2)$.

6. Find $\lim\limits_{x \to -1} \dfrac{-1}{(1-x)^3}$.

7. Define vertical asymptote.

8. $\dfrac{d}{dx} \int\limits_{-\pi}^{x} \cos t\, dt = \text{.......}$

9. Express the limit $\lim\limits_{\|P\| \to 0} \sum\limits_{k=1}^{n} \dfrac{1}{1-c_k} \Delta x_k$ where P is the partition of $[2, 3]$ as an integral.

10. State Mean Value Theorem.

Turn over

11. Find all possible functions with derivative $y' = x^3$.

12. Shortest interval length of a partition is called —————.

$(12 \times 1 = 12 \text{ marks})$

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Show that if $\lim\limits_{x \to c} |f(x)| = 0$, then $\lim\limits_{x \to c} f(x) = 0$.

14. If $\sqrt{5 - 2x^2} \le f(x) \le \sqrt{5 - x^2}$ for $-1 \le x \le 1$, find $\lim\limits_{x \to 0} f(x)$.

15. If $\lim\limits_{x \to -2} \dfrac{f(x)}{x^2} = 1$, find $\lim\limits_{x \to -2} \dfrac{f(x)}{x}$.

16. Find the slope of the curve $y = 1/x$ at $x = a$.

17. Differentiate $f(x) = \dfrac{x}{x-1}$. Where does the curve $y = f(x)$ have slope -1 ?

18. Show that $y = -x-$ is not differentiable at $x = 0$.

19. Find the equation for the tangent to the curve $y = x + \dfrac{2}{x}$ at $(1, 3)$.

20. Find absolute extrema of $y = x^{2/3}$ on $[-2, 3]$.

21. Find the function whose derivative is $\sin x$ and whose graph passes through the point $(0, 2)$.

22. Show that $\lim\limits_{x \to \infty} \dfrac{1}{x} = 0$.

23. Show that if f is continuous on $[a, b]$, $a \ne b$ and if $\int_a^b f(x)\, dx = 0$, then $f(x) = 0$ at least once in $[a, b]$.

24. Express the solution of the initial value problem $\dfrac{dy}{dx} = \tan x$, $y(1) = 5$ as an integral.

$(9 \times 2 = 18 \text{ marks})$

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 marks.

25. Show that the line $y = mx + b$ is its own tangent at any point $(x_0, mx_0 + b)$.

26. When does a function not have a derivative at a point ? Explain.

27. Show that if f has a derivative at $x = c$, then f is continuous at $x = c$.

28. Show that functions with same derivatives differ by a constant.

29. Find the asymptotes of the curve $y = \dfrac{x+3}{x+2}$.

30. Find $\lim\limits_{x \to \infty} \dfrac{2\sqrt{x} + x^{-1}}{3x - 7}$.

31. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \le x \le 2$ about the x-axis.

32. Find $\lim\limits_{x \to 0^+} \dfrac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$.

33. Define $f(3)$ in a way that extends $f(x) = \dfrac{x^2 - 9}{x - 3}$ to be continuous at $x = 3$.

$(6 \times 5 = 30 \text{ marks})$

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 marks.

34. Find the critical points of $f(x) = x^{1/3}(x - 4)$. Identify the intervals on which f is increasing and decreasing. Find the function's local and absolute extreme values.

35. Find the volume of the solid generated by revolving the regions bounded by the curve $x = \sqrt{5}\, y^2, x = 0, y = -1, y = 1$ about x-axis.

Turn over

36. Let $f(x) = \begin{cases} \sqrt{1-x^2}, & 0 \le x < 1; \\ 1, & 1 \le x < 2; \\ 2, & x = 2. \end{cases}$

(a) What are the domain and range of f ?

(b) At what points c, if any, does $\lim_{x \to c} f(x)$ exist ?

(c) At what points does only the left-hand limit exists ?

(d) At what points does only the right-hand limit exists ?

$(2 \times 10 = 20 \text{ marks})$