D 43254 (Pages : 3) Name.....................................

Reg. No..................................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2018

(CUCBCSS)

Complementary Course

STS 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in* **one word.**

Each question carries 1 *mark.*

Name the following :

1. The moments of a random variable X about origin.
2. The probability distribution in which mean is equal to its variance.
3. The distribution of $\frac{x_1 - x_2}{\sqrt{2}}$ where $X_1 \sim N(1,1)$ and $X_2 \sim N(1,1)$.

Fill up the blanks :

4. If two variables X and Y are independent, then $E(XY) =$ ————.
5. The maximum height of the normal curve lies at the point ————.
6. The mode of the geometric distribution $f(x) = \left(\frac{1}{2}\right)^x$; $x = 1, 2, ...$ is ————.
7. If $X \sim N(12.5, 12.25)$ and $Y \sim N(8.5, 6.25)$, the variable X + Y is distributed as ————.

Write True *or* False :

8. If X and Y are two random variables, then the covariance between the variables $aX + b$ and $cY + d$ is equal to covariance between X and Y.
9. For a binomial distribution mean is always less than the variance.
10. Convergence in probability is also known as weak convergence.

(10 × 1 = 10 marks)

Turn over

Section B

Answer **all** *questions in* **one sentence** *each.*
Each question carries 2 *marks.*

11. Define variance of a random variable.

12. Give the properties of characteristic function.

13. Define conditional expectation.

14. Define joint central moments for the bivariate distribution.

15. Define negative binomial distribution.

16. If a random variable $X \sim N\left(40, 5^2\right)$, find $P\left(45 \le X \le 50\right)$.

17. Define weak convergence.

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each question carries 4 *marks.*

18. Define moment generating function of a random variable. Prove that it does not exist always.

19. Give the properties of normal distribution.

20. If X and Y are independent Poisson variates, show that the conditional distribution of $\left(X \mid X + Y\right)$ is binomial.

21. State the weak law of large numbers and central limit theorem.

22. If $E\left(X\right) = 5$, $V\left(X\right) = 3$ and if $P\left[\left|X - 5\right| < h\right] \ge 0.99$, find the value of h.

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each question carries 6 *marks.*

23. A coin is tossed until a head appears. What is the expectation of the number of tosses required ?

24. X_1 and X_2 have a bivariate distribution given by $p(x_1, x_2) = \frac{x_1 + 3x_2}{24}$; where $(x_1, x_2) = (1,1), (1,2),$ $(2,1), (2,2)$. Find the conditional mean and conditional variance of X_1 given $X_2 = 2$.

25. Let the random variable X assumes the value 'x' with the probability law $P(X = x) = q^{x-1}p$; $x = 1, 2, 3.....$ and $q = 1 - p$. Find the m.g.f, of X and hence find its mean and variance.

26. The mean and variance of a binomial distribution are $\frac{8}{3}$ and $\frac{16}{9}$. Find (i) $P(X = 1)$ and (ii) $P(X \leq 1)$.

27. Assuming that the height of students is distributed as $N(\mu, \sigma^2)$. Out of a large number of students, 5% are under 72 inches and 10% are below 60 inches. Find the values of μ and σ.

28. Examine whether the weak law of large numbers holds $\{X_k\}$ of independent random variables defined as follows :

$$P\left[X_k = \pm 2^k\right] = 2^{-(2k+1)} \text{ and } P[X_k = 0] = 1 - 2^{-2k}.$$

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each question carries 10 *marks.*

29. Let X and Y be two random variables, prove that :

 (i) $E(X) = E\{E(X|Y)\}$ and

 (ii) $V(X) = E\{V(X|Y)\} + V\{E(X|Y)\}$.

30. State and prove the recurrence relation for central moments for a binomial distribution.

31. Derive the m.g.f. of a normal distribution with parameters μ and σ^2.

32. State and prove the Chebychev's inequality.

(2 × 10 = 20 marks)