D 43194 **(Pages : 3)** **Name.......................................**

Reg. No...................................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2018

(CUCBCSS—UG)

Mathematics

MAT 2B 02—CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*
Each question carries 1 mark.

1. Absolute maximum of the function $y = x^2$ on $(0, 2]$ is ....

2. Find dy if $y = x^5 + 37x$.

3. Find the interval in which the function $y = x^3$ is concave up.

4. Suppose that $\int_1^4 f(x)\, dx = -2$, evaluate $\int_4^1 f(x)\, dx$.

5. A partition's longest subinterval is called ————.

6. Find $\lim\limits_{x \to -\infty} \dfrac{\pi\sqrt{3}}{x^2}$.

7. Express the limit of Riemann sums $\lim\limits_{\|P\| \to 0} \sum\limits_{k=1}^{n} \left(3c_k^{\,2} - 2c_k + 5\right) \Delta x_k$ as an integral if P denotes a partition of the interval $[-1, 3]$.

8. Find the norm of the partition $[0, 1.2, 1.5, 2.3, 2.6, 3]$.

9. Define critical point of a function.

10. Evaluate $\int 5\sec x \tan x\, dx$.

11. State Rolls' Theorem.

12. Define point of inflection.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Evaluate $\lim_{x \to \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$.

14. Find the absolute extrema of $h(x) = x^{2/3}$ on $[-2, 3]$.

15. Find the interval in which $f(t) = -t^2 - 3t + 3$ is increasing and decreasing.

16. Find dy/dx if $y = \int_1^{x^2} \cos t \, dt$.

17. Suppose $\int_1^x f(t)\, dt = x^2 - 2x + 1$. Find $f(x)$.

18. Evaluate $\sum_{k=1}^{4} \left(k^2 - 3k\right)$.

19. Give an example of a function with no Riemann integral. Explain.

20. Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point $(0, 2)$.

21. Use Max-Min inequality to find upper and lower bounds for the value of $\int_0^1 \frac{1}{1+x^2}\, dx$.

22. Show that the value of $\int_0^1 \sqrt{1 + \cos x}\, dx$ cannot possibly be 2.

23. Find the linearization of $f(x) = \cos x$ at $x = \pi/2$.

24. Suppose that $F(x)$ is an antiderivative of $f(x) = \frac{\sin x}{x}, x > 0$. Express $\int_1^3 \frac{\sin 2x}{x}\, dx$ in terms of F.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Find the linearization of $f(x) = 2 - \int_2^{x+1} \frac{9}{1+t}\,dt$ at $x = 1$.

26. Find the area of the region between the curve $y = x^2$ and the x-axis on the interval $[0, b]$.

27. Find the asymptotes of the curve $y = 2 + \frac{\sin x}{x}$.

28. A rectangle is to be inscribed in a circle of radius 2. What is the largest area the rectangle can have, and what are its dimensions ?

29. Show that functions with zero derivatives are constant.

30. Find the lateral surface area of the cone generated by revolving the line segment $y = x/2, 0 \le x \le 4$, about the x-axis.

31. Show that if f is continuous on $[a, b, a \neq b$, and if $\int_a^b f(x)\,dx = 0$, then $f(x) = 0$ atleast once in $[a, b]$.

32. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by x-axis and the line $y = x - 2$.

33. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \le x \le 2$, about the x-axis.

(6 × 5 = 30 marks)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. (a) Find the curve through the point $(1, 1)$ whose length integral is $L = \int_1^4 \sqrt{1 + \frac{1}{4x}}\,dx$.

    (b) How many such curves are there ?

35. Find the length of the curve $y = (1/3)\left(x^2 + 2\right)^{3/2}$ from $x = 0$ to $x = 3$.

36. Find the volume of the solid generated by revolving the regions bounded by the curve $x = \sqrt{5y^2}, x = 0, y = -1, y = 1$ about x-axis.

(2 × 10 = 20 marks)