D 51243 (Pages : 4) Name.......................................

Reg. No...................................

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS—UG)

Core Course

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*

1. Find $\frac{d}{dx} \ln 2x$.
2. Define a sequence.
3. Find least upper bound of $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\ldots, \frac{n}{n+1}$.
4. Find a formula for n^{th} term of the sequence $1, 5, 9, 13, 17, \ldots.$
5. State Sandwich theorem for sequences.
6. If $|r| < 1$ the series $a + ar + ar^2 + \ldots + ar^{n-1} + \ldots.$ converges to............
7. Define conditional convergence of a series.
8. Write a parametrization of the circle $x^2 + y^2 = 1$.
9. $\lim_{n \to \infty} \sqrt[n]{n} =$..................
10. Write the polar form of the parabola $y^2 = Qax$.
11. Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$. If $\lim_{n \to \infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$..........
12. If $\sum |a_n|$ is convergent, then $\sum a_n$ is ....

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. Find $\int_{-\pi/2}^{\pi/2} \frac{4\cos\theta}{3+2\sin\theta}\, d\theta.$

14. Find k if $e^{2k} = 10.$

15. Find $\int_{0}^{\ln 2} e^{3x}\, dx.$

16. Show that $\lim_{n \to \infty} \frac{1}{n} = 0.$

17. For what values of x do the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converge ?

18. Find the series for $f'(x)$ and $f''(x)$ if $f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, -1 < x < 1.$

19. Find the focus and directrix of the parabola $y^2 = 10x.$

20. Find the eccentricity of the hyperbola $x^2 - y^2 = 1.$

21. Determine the conic section from the equation $xy - y^2 - 5y + 1 = 0.$

22. Graph the sets of points whose polar co-ordinates satisfy the conditions $-3 \le r \le 2$ and $\theta = \pi/2.$

23. Replace the polar equation $r^2 = 4r\cos\theta$ by equivalent Cartesian equation.

24. Find the equation for the hyperbola with eccentricity $3/2$ and directrix $x = 2.$

$(9 \times 2 = 18$ marks)

Part C (Short Essay Type)

Answer any **six** *questions.*

25. Solve the initial value problem $e^y \dfrac{dy}{dx} = 2x, x > \sqrt{3}, y(2) = 0$.

26. Show that $(-1)^{n+1} \dfrac{n-1}{n}$ diverges.

27. Find a formula for the n^{th} partial sum of the series $\dfrac{1}{2 \cdot 3} + \dfrac{1}{3 \cdot 4} + \dfrac{1}{4 \cdot 5} + .. + \dfrac{1}{(n+1)(n+2)} + ...$ and use it to find the series sum if it converges.

28. Identify the function $f(x) = x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + ..., -1 \leq x \leq 1$.

29. The x and y axes are rotated through an angle of $\pi/4$ radians about the origin. Find an equation for the hyperbola $2xy = 9$ in the new co-ordinates.

30. Find the surface area generated by revolving the curves $x = \cos t, y = 2 + \sin t, 0 \leq t < 2\pi$ about x-axis.

31. Show that $(1/2, 3\pi/2)$ lies on the curve $r = -\sin(\theta/3)$.

32. Determine whether the series $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$ converges or diverges.

33. Check whether $\sum_{n=2}^{\infty} \dfrac{1 + n \ln n}{n^2 + 5}$ converges or diverges.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*

34. The series $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$ converges to $\sin x$ for all x.

 (a) Find the first six terms of the series for cos x. For what values of x should the series converge ?

 (b) By replacing by $2x$ in the series for sin x, find a series that converges to sin $2x$ for all x.

35. Find the Taylor series and Taylor polynomials generated by $f(x) = \cos x$ at $x = 0$.

36. Find the length of the curve curve $x = 8\cos t + 8t \sin t$, $y = 8\sin t - 8t \cos t$, $0 \leq t \leq \pi/2$.

(2 × 10 = 20 marks)