D 41959 **(Pages : 4)** **Name..................................**

Reg. No...............................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2018

(CUCBCSS—UG)

Mathematics

MAT 4B 04—THEORY OF EQUATIONS MATRICES AND VECTOR CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer **all the twelve** *questions.*
Each question carries 1 *mark.*

1. If α, β, γ are the roots of $2x^3 + 3x^2 - x - 1 = 0$. Find the equation whose roots are $\frac{1}{2\alpha}, \frac{1}{2\beta}, \frac{1}{2\gamma}$.

2. State the Fundamental theorem of algebra.

3. If α, β, γ are the roots of $ax^3 + 3bx^2 + 3cx + d = 0$. Find the value of :

   $(\beta - \gamma)(\gamma - \alpha) + (\gamma - \alpha)(\alpha - \beta) + (\alpha - \beta)(\beta - \gamma)$.

4. What do you mean by reciprocal equation of second type ? Give example.

5. What is the rank of the identity matrix of order 20 ?

6. If $A = [a_{ij}]$ is an $m \times n$ matrix and $a_{ij} = 7$ for all i, j then rank of A is ————.

7. A system of m homogeneous linear equations in n unknowns has only trivial solution if ————.

8. For what values of a the system of equations $ax + y = 1, x + 2y = 3, 2x + 3y = 5$ are consistent.

9. If the number of variables in a non-homogeneous system AX = B is n then the system possesses a unique solution if ————.

10. Find the parametric equation of a line through P (3, – 4, – 1) and parallel to the vector $i + j + k$.

11. Find the unit vector tangent to the curve $r(t) = \left(\cos^3 t\right) j + \left(\sin^3 j\right) k, 0 \le t \le \frac{\pi}{2}$.

12. Write equations relating rectangular and cylindrical co-ordinates.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

*Answer any **nine** questions.*
Each question carries 2 marks.

13. Solve $4x^3 - 24x^2 + 23x + 18 = 0$. Given that the roots are in arithmatic progression.

14. Transform $x^3 - 6x^2 + 5x + 12 = 0$ into an equation lacking second term.

15. If α, β, γ are the roots of $x^3 + qx + r = 0$. Find the equation whose roots are $(\beta - \alpha)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$.

16. If $A = \begin{pmatrix} -2 & -1 \\ 5 & 4 \end{pmatrix}$. Find A^{-1}.

17. Prove that the characteristic roots of Hermitian matrix are real.

18. If α is an eigen value of a non-singular matrix A, prove that $\frac{|A|}{\alpha}$ is an eigen value of adj A.

19. Show that the product of characteristic roots of a square matrix of order n is equal to the determinant of the matrix.

20. Find the value of a for which r (A) = 3 where $A = \begin{pmatrix} 2 & 4 & 4 \\ 3 & 1 & 2 \\ 1 & 0 & a \end{pmatrix}$.

21. Find the velocity and acceleration vectors of $r(t) = (3\cos t)\, i + (3\sin t)\, j + t^2 k$.

22. Find a Cartesian equation for the surface $z = r^2$. And identify the surface.

23. Evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left[(\sin t)\, i + (1 + \cos t)\, j + \left(\sec^2 t\right) k\right] dt$.

24. Find the normal vector for $r(t) = (a \cos t)\, i + (a \sin t)\, j + bk$.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. If α, β, γ are the roots of $x^3 - x - 1 = 0$. Find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$.

26. Solve the equation $x^2 - 12x - 65 = 0$ by Cardan's method.

27. Solve $x^3 + 6x^2 + 3x + 18 = 0$.

28. Prove that the rank of a non-singular matrix is equal to the rank of its reciprocal matrix.

29. Find the rank of $\begin{pmatrix} 4 & -2 & 3 \\ 2 & 0 & 1 \\ 3 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$.

30. Using matrix method solve :

$$\begin{aligned} x + 2y + z &= 2 \\ 3x + y - 2z &= 1 \\ 4x - 3y - z &= 3 \\ 2x + 4y + 2z &= 4. \end{aligned}$$

31. Find the point in which the line $x = 1 - t, y = 3t, z = 1 + t$ intersects the plane $2x - y + 3z = 6$.

32. Find the distance from the point S (0, 0, 1, 2) to the line $x = 4t, y = -2t, z = 2t$.

33. Find the eigen values and eigen vectors of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve the equation $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.

35. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \\ 0 & 5 & -1 \end{pmatrix}$ and hence evaluate A^{-1}.

36. Find the binormal vector and torsion for the space curve $r(t) = \left(e^t \cos t\right) i + \left(e^t \sin t\right) j + 2k$.

(2 × 10 = 20 marks)