

## FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2019

(CUCBCSS-UG)

Mathematics

MAT 4C 04—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

## Part A (Objective Type)

*Answer all the twelve questions.**Each question carries 1 mark.*

1. State the Existence and Uniqueness theorem for initial value problem.
2. Define and give an example of an even function.
3. What do you mean by a non-linear differential equation ?
4. Solve  $y'' - y' - 2y = 0$ .
5. Define a unit step function.
6. State the existence theorem for Laplace transforms.
7. Find  $L^{-1}\left(\frac{a}{s^2 - a^2}\right)$ .
8. Find  $L\left(t^{-1/2}\right)$ .
9. Define a rectangular wave.
10. Write the 2-dimensional Poisson equation.
11. Give a formula for an error for Simpson's rule.
12. Write the formula for Runge Kutta method.

(12 × 1 = 12 marks)

## Part B (Short Answer Type)

*Answer any nine questions.**Each question carries 2 marks.*

13. Find the particular integral for  $y' + 4y = 8x^2$ .
14. Find a basis for the solution of the differential equation  $y'' + y = 0$ .

Turn over

15. If  $L^{-1}(f(s)) = F(t)$  then show that  $L^{-1}(f(s-a)) = e^{at}F(t)$ .
16. Solve  $3y'' - 8y' - 3y = 0, y(-3) = 1, y(3) = \left(\frac{1}{e^2}\right)$ .
17. Find  $L(e^{-\alpha t} \cos \beta t)$ .
18. If  $f(x)$  is a periodic function of  $x$  of period  $p$ , show that  $f(ax), a \neq 0$ , is a periodic function of  $x$  of period  $\frac{p}{a}$ .
19. Find the Fourier series of  $f(x) = x + |x|, -\pi < x < \pi$ .
20. Show that  $u = e^{-t} \sin x$  is a solution of heat equation.
21. Apply Picards iteration to solve  $y' = y - x^2, y(0) = 1$  also find  $y(0.1)$  and  $y(0.2)$ .
22. Evaluate  $\int_{-3}^3 x^4 dx$  using Simpson's rule.
23. What do you mean by convolution ?
24. Evaluate  $\int_0^6 \frac{1}{1+x} dx$  by Trapezoidal rule.

(9 × 2 = 18 marks)

**Part C (Short Essays)**

*Answer any six questions.  
Each question carries 5 marks.*

25. Solve  $(4x^2D^2 + 12xD + 3)y = 0$ .
26. Find a general solution of the differential equation  $y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$ .
27. Find the Laplace transform of  $(t-1)^2 u(t-1)$ .

28. Find  $L^{-1} \left( \frac{4(e^{-2s} - 2e^{-5s})}{s} \right)$ .
29. Solve  $u_{xy} = u_x$ .
30. Find the Fourier series of  $f(x) = \begin{cases} -k, & \text{if } -\pi < x < 0; \\ k, & \text{if } 0 < x < \pi, \end{cases}$  and  $f(x+2\pi) = f(x)$ .
31. Given  $y' = -y, y(0) = 1$ . Find the value of  $y$  at  $x = (0.01)(0.01)(0.04)$  by improved Euler method.
32. Find approximate solution to  $y' + y = e^x, y(0) = 0$ .
33. Evaluate  $\int_4^{5.2} \log_e x \, dx$  using Simpson's rule.

(6 × 5 = 30 marks)

**Part D**

*Answer any two questions.  
Each question carries 10 marks.*

34. Solve  $x^2 y'' - 2xy' + 2y = x^3 \sin x$ .
35. Solve the integral equation  $y(t) = t + \int_0^t y(\tau) \sin(t - \tau) d\tau$ .
36. Find the Fourier series  $f(x) = \begin{cases} x + x^2, & \text{if } -\pi < x < \pi; \\ \pi^2, & \text{if } x = \pm \pi. \end{cases}$

(2 × 10 = 20 marks)