C 30799 (Pages : 3) Name..................................

Reg. No..................................

FIFTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION, NOVEMBER 2017

(UG–CCSS)

MM 5B 05—VECTOR CALCULUS

Time : Three Hours Maximum : 30 Weightage

I. Answer *all* questions :

1 Find the curl of $F(x, y) = (x^2 - y)i + (xy - y^2)j$.

2 Examine whether $F = yzi + zxj + xyk$ is conservative.

3 If $\vec{F}$ is a field defined on D and $\vec{F} = \nabla f$ for some scalar function f on D then f is called a ———— of $\vec{F}$.

4 Find the gradient field of $g(x, y, z) = e^z - \ln(x^2 + y^2)$.

5 Define gradient field of a differentiable function.

6 Define critical point.

7 Say True *or* False :

If a function $f(x, y)$ is differentiable at (x_0, y_0) then f is continuous at (x_0, y_0).

8 Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x, y) = x^2 + 3xy + y - 1$ at $(4, -5)$.

9 $\lim_{(x,y)\to(3,4)} \sqrt{x^2 + y^2 - 1} =$ ————.

10 If $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ then $f(3, 0, 4) =$ ————.

11 If $\vec{r}(t) = (\cos t)i + (\sin t)j + tk$ then $\lim_{t\to\frac{\pi}{4}} \vec{r}(t) =$ ————.

12 Find the parametric equation for the line through P (1, 2, – 1) and Q (– 1, 0, 1).

(12 × ¼ = 3 weightage)

II. Answer *all* nine questions :

13 Find the curl of $F = (x^2 - y)i + 4zj + x^2k$.

14 State Green's theorem (normal form).

Turn over

15 Evaluate $\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}(x+y+z)\,dy\,dx\,dz$.

16 Write an equivalent integral for $\int_{0}^{1}\int_{2}^{4-2x} dy\,dx$ with the order of integration reversed.

17 Find the direction in which $f(x, y) = x^2 + xy + y^2$ increases most rapidly at $p_0(-1, 1)$.

18 If $w = x^2 + y^2 + z^2$ and $z = x^2 + y^2$ find $\left(\frac{\partial w}{\partial y}\right)_z$.

19 If $w = xy + \frac{e^y}{y^2+1}$ find $\frac{\partial^2 w}{\partial x\,\partial y}$.

20 Write the range of the function $f(x, y) = xy$.

21 Find the length of one turns of the helix $\vec{r}(t) = (\cos t)\,i + (\sin t)\,j + tk$.

(9 × 1 = 9 weightage)

III. Answer any *five* questions :

22 Evaluate $\iint_R e^{x^2+y^2}\,dy\,dx$ where R is the semi-circular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$.

23 Find the local extreme values of the function $f(x, y) = xy - y^2 - x^2 - 2x - 2y + 4$.

24 Find $\frac{dw}{dt}$ at $t = \pi$. Given $w = x^2 + y^2$, $x = \cos t$, $y = \sin t$.

25 Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$.

26 Show that $f(x, y) = \begin{cases} \frac{2xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0 \quad , & (x, y) = (0,0) \end{cases}$ is discontinuous at the origin.

27 Find the torsion for the space curve $\vec{r}(t) = (3\sin t)\,i + (3\cos t)\,j + 4t\,k$.

28 Find T and N for the plane curve $\vec{r}(t) = t\,i + (\ln\cos t)\,j,\ -\frac{\pi}{2} < t < \frac{\pi}{2}$.

(5 × 2 = 10 weightage)

IV. Answer any *two* questions :

29 Find the area of the cap cut from the hemisphere $x^2 + y^2 + z^2 = 2, z \geq 0$, by the cylinder $x^2 + y^2 = 1$.

30 Find the work done by $F = (y - x^2)i + (z - y^2)j + (x - z^2)k$ over the curve $\vec{r}(t) = ti + t^2 j + t^3 k, 0 \leq t \leq 1$ from (0, 0, 0) to (1, 1, 1).

31 Find an upper bound for the magnitude of the error E in the approximation $f(x, y) \approx L(x, y)$ over the rectangle R. Given $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$, $P_0(3, 2)$. $R : |x - 3| \leq 0.1, |y - 2| \leq 0.1$.

(2 × 4 = 8 weightage)