D 50601 **(Pages : 3)** **Name.................................**

Reg. No..............................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS—UG)

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Part A

Answer **all** *the twelve questions.*
Each question carries 1 *mark.*

1. Fill in the blanks : Supremum of the set $S = \{1/m - 1/n \, ; m, n \in N\}$ is ————.
2. Determine the set $A = \{x \in R : |2x + 3| < 7\}$.
3. The Set of all real numbers which satify the inequality $0 \le a < \in, \forall \in > 0,$ then a = ————.
4. Fill in the blanks : The Supremum property of R states that ————.
5. State the Trichotomy Property of R.
6. Give the condition for a subset of R to be an Interval of R.
7. State the general Arithmetic-Geometric mean Inequality of real numbers .
8. Fill in the blanks : The Characterization Theorem of Open sets states that ————.
9. State the Archimedian Property of positive Integers.
10. If $c > 0$, then $\lim (c^{1/n}) =$ ————.
11. State the Bolzano-Weierstrass Theorem on Sequence .
12. Fill in the blanks : The Exponential form of $-1-i =$ ————.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. State and prove Bernoulli's Inequality of Real numbers.
14. Show that there doesnot exist a rational number r such that $r^2 = 2$.
15. If $a, b \in R$, then prove that $\| a | - | b \| \le | a - b \|$.

Turn over

16. Prove that a real sequence can have atmost one limit.

17. If $x \in$ R then prove that there exists $n_x \in$ N such that $x < n_x$.

18. State and prove the "Betweeness Property" of Irrational numbers.

19. If X = (x_n) is a convergent sequences of real numbers satisfying lim $(x_n) = x$, then show that $\lim(|x_n|) = |x|$.

20. Prove that the set of irrational numbers is uncountable.

21. Let A and B be bounded non-empty subsets of real numbers such that $a \leq b, \forall a \in A, b \in B$. Prove that Sup A $\leq$ Inf B.

22. Discuss the convergence of X = (x_n) defined by $x_n = n$, if n odd and $x_n = 1/n$, if n even.

23. Show that every bounded sequence of real numbers has a converging sub-sequence.

24. Test the convergence of the sequence $\left(\frac{\sin n}{n}\right)$.

25. Define Cauchy sequence test whether $(1/n)$ is a Cauchy sequence or not.

26. Find all values of $(-8i)^{1/3}$.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*
Each question carries 7 marks.

27. State and prove the Nested Interval Property.

28. Define denumerable set. Show that the set Q of rational numbers is denumerable.

29. If the set A_m is countable for each m $\in$ N, then prove that $A = \bigcup_{m=1}^{m=\infty} A_m$ is countable.

30. X = x_n and Y = y_n be sequences of real numbers converges to x and y respectively, then prove that X.Y converges to xy.

31. (a) Give an example of a convergent sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n}\right) = 1$.

(b) Give an example of a dververgent sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n}\right) = 1$.

(c) Give your comments about the property of the sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n}\right) = 1$.

32. If $X = (x_n)$ is a real sequence and $X_m = (x_{m+n} : n \in N)$ is the m-tail of X ; $m \in N$, then show that X_m converges to x if and only if X converges to x.

33. Find a sequence (x_n) of real numbers such that $\lim |x_{n+1} - x_n| = 0$, but not a Cauchy sequence.

34. (a) Find the Arg Z, if $Z = \frac{i}{-2-2i}$.

(b) Express the complex number $\left(\sqrt{3+i}\right)^7$ in Rectangular form.

35. Discuss the convergence of the following sequences whose n'th terms are defined by $(a)\ x_n = \left(1+\frac{1}{n^2}\right)^{2n^2}$ and $(b)\ y_n = \frac{\log n}{n}$.

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. State and prove the Cauchy convergence criterion for sequence.

37. (a) Show that the unit interval [o,1] is uncountable.

(b) State and prove the Ratio Test for the convergence of real sequence.

38. (a) Define closed sets in R. Show that the Intersection of an arbitrary collection of closed sets in R is closed.

(b) Show by an example that the union of infinitely many closed sets in R need not be closed.

(2 × 13 = 26 marks)