D 50600 **(Pages : 3)** **Name..................................**

Reg. No...............................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS—UG)

MAT 5B 06—ABSTRACT ALGEBRA

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Define subgroup of a group.
2. Fill in the blanks : The units in the ring of integers $\mathbb{Z}$ are ————.
3. Write the order of the permutation (1, 2) (1 9 8) in S_9.
4. Give an example of a finite group of order 4 which is not cyclic.
5. Calculate the remainder obtained when 45^{72} is divided by 73.
6. Compute (1, 4) (7, 5) (2, 5, 7) in S_7.
7. What is the characteristic of the ring $< \mathbb{Z}_9, +_9, \times_9 >$.
8. Give an example for an integral domain which is not a field.
9. What is the necessary condition for a homomorphism ϕ from a group G to G' to be injective.
10. Write two equivalent conditions for the subgroup of a group to be a normal subgroup.
11. What is the index of A_n in S_n.
12. Denne a cyclic group and give an example.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 *marks.*

13. Let S be a set and let *f*, *g* and *h* be functions mapping S into S. Prove that $(fog)\,oh = fo\,(goh)$.
14. Show that every group of prime order is abelian.
15. Draw the group table for S_3.

Turn over

16. Define * by $a * b = \frac{ab}{3}$ then show that Q^+, the set of positive rationals, with operation * s a group.

17. Show that A_n is a normal subgroup of S_n and find a group to which S_n/A_n is isomorphic.

18. Define a field. Show that $(a + b)^2 = a^2 + 2ab + b^2$ in any field using the axioms of the field.

19. Give an example of non-commutative finite ring. Establish that it is so.

20. Give any necessary and sufficient condition for a ring R to have no zero divisors. Justify your claim.

21. Find a formula for identifying units in the ring of guassian integers $\{a + ib : a, b \in \mathbb{Z}\}$.

22. Write all the left cosets of $3\mathbb{Z}$ in $\mathbb{Z}$.

23. Show that factor group of a cyclic group is cyclic.

24. Solve : $x^2 = i$ in S_3 where i is the identity.

25. Define group homomorphism and state fundamental theorem of homomorphism.

26. Is Q, the set of rationals, the field of quotients for integers ? Justify your claim.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 marks.

27. Show that the binary structure $< \mathbb{R}, + >$ with operation the usual addition is isomorphic to the structure $< \mathbb{R}^+, . >$ where is the usual multiplication.

28. Define order of an element in any group G. Show that in a finite group G, order of any element divides order of G.

29. Show that the subset S of $M_n(\mathbb{R})$ consisting of all invertible $n \times n$ matrices under matrix multiplication is a group.

30. Show that every permutation σ of a finite set is a product of disjoint cycles.

31. Let G and G′ be groups and let $\Phi : G \rightarrow G'$ be one to one function such that $\Phi(xy) = \Phi(x)\,\Phi(y)$ for all $x, y \in G$. Then prove that $\Phi[G]$ is a subgroup of G' and Φ provides an isomorphism of G with $\Phi[G']$.

32. Show that every proper subgroup of a group G, with o(G) = pq where p and q are prime, is cyclic.

33. Solve the equation : $x^2 - 5x + 6 = 0$ completely in $\mathbb{Z}_{12}$.

34. Establish a formula for computing the number of zero divisors in the ring $\mathbb{Z}_n$ by giving its proof.

35. Give a necessary and sufficient condition for union of two subgroups of a group to be a subgroup.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*

Each question carries 13 *marks.*

36. (a) Find the index of the subgroup generated by $\sigma = (1, 2, 5, 4)\,(2, 3)$ in S_5.

    (b) Write all the subgroups of Z_{10}.

37. State and prove Cayley's theorem in detail.

38. (a) List all the units in the matrix ring $M_2(\mathbb{Z}_2)$.

    (b) Show that every finite integral domain is a field.

(2 × 13 = 26 marks)