VISCOSITY

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Streamline flow

- Slow and steady flow
- Liquid flow-flow of different layers
- In a layer- all the particle has same velocity
- Different layers- velocities different

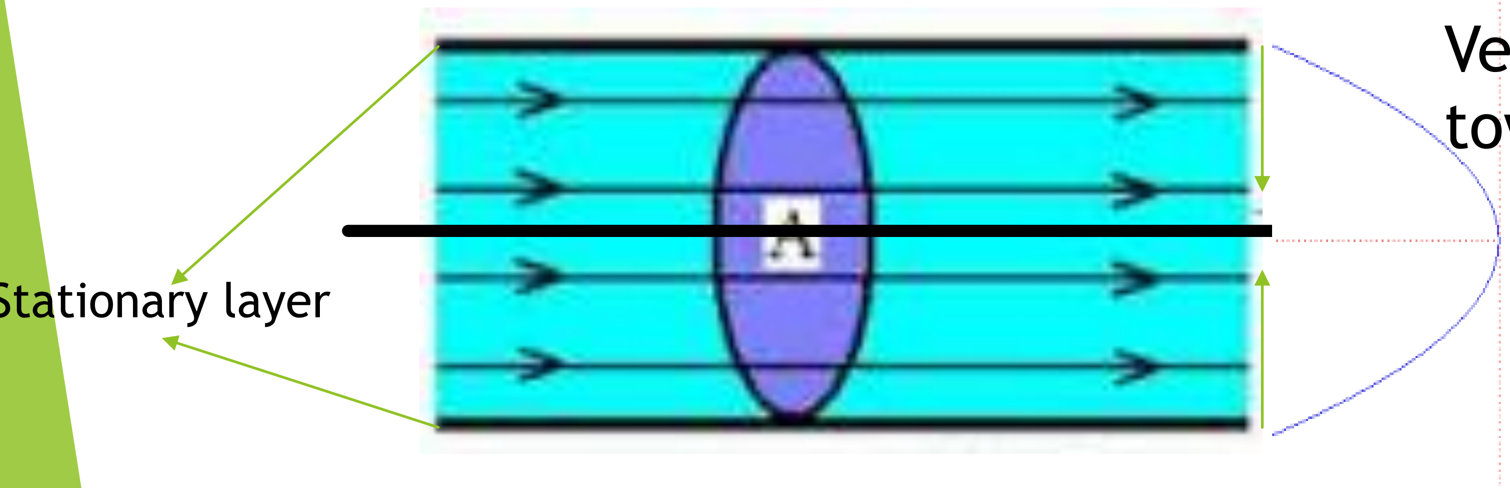
Velocities of layers increases as distance from the fixed surface increases

Velocity Gradient $= \frac{Change\ of\ velocity}{distance} = \frac{dv}{dx}$

Layer in contact with fixed surface- stationary

Liquid flow in a tube

- The layers are coaxial cylindrical shells
- Layer in contact with the tube is stationary
- Velocity increases towards the axis

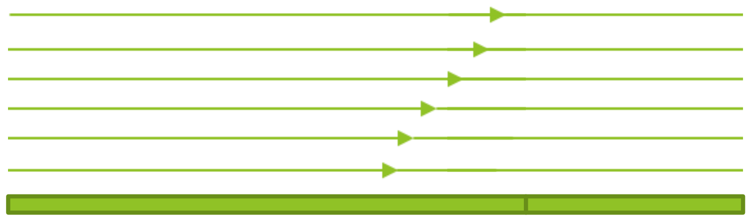


Velocities of layers increases towards the axis

Viscosity

- Any layer- retarded by below layer

  - accelerated by above layer



- Net tangential force which opposes the motion

- Viscosity – Property of a liquid by virtue of which it opposes the relative motion between its different layers

Coefficient of Viscosity

- According to Newton,

  Viscous force $\propto area \times velocity\ gradient$

$$F \propto A\frac{dv}{dx}$$

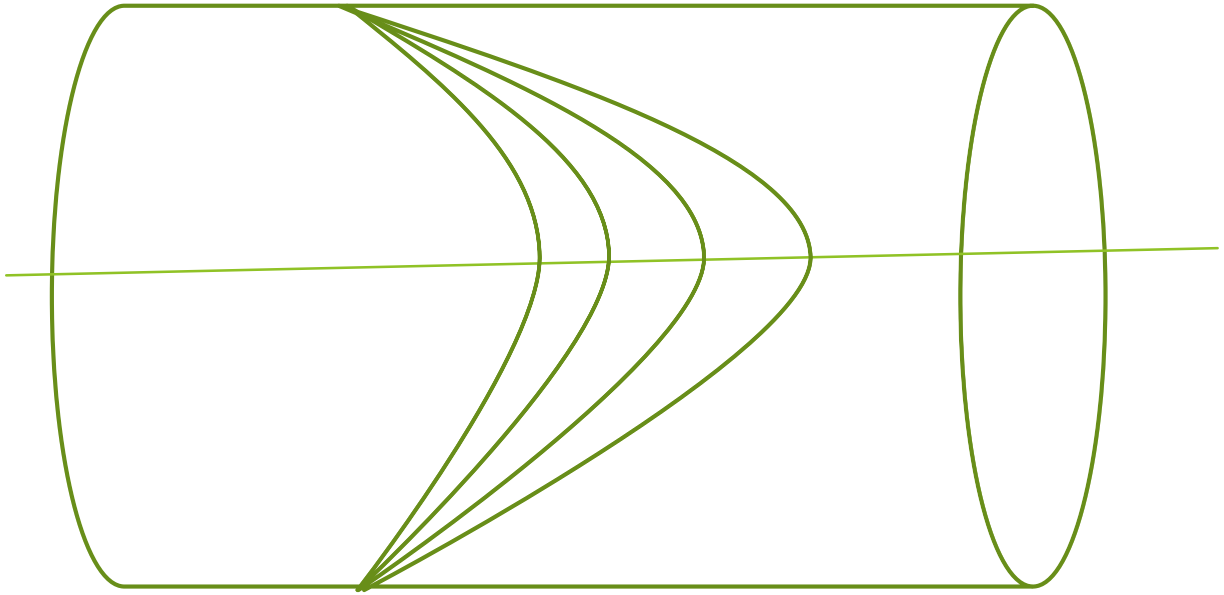
$$F = -\eta A\frac{dv}{dx}$$ (Newton's law of viscous flow in stream line motion)

- Negative sign-viscous force is opposite to velocity
- When $A = 1, \frac{dv}{dx} = 1, \qquad |F| = \eta$
- Coefficient of Viscosity- Tangential force per unit area to maintain unit velocity gradient between the layers of the liquid

Unit & Dimension

- $F = -\eta A \frac{dv}{dx}$ $\eta = -\frac{F}{A\frac{dv}{dx}}$
- Unit of η $=\frac{N}{m^2\frac{ms^{-1}}{m}} = \frac{N}{m^2 s^{-1}} = \frac{Ns}{m^2}$=Pascal second=Poiseuille
- Dimension of η $=\frac{MLT^{-2}}{L^2\frac{LT^{-1}}{L}} = \frac{MLT^{-2}}{L^2 T^{-1}} = ML^{-1}T^{-1}$

Poiseuille's Equation
Rate of flow of a liquid through a tube



- Consider a liquid of

  Coefficient of viscosity η,

flowing through a tube of

  Length-l

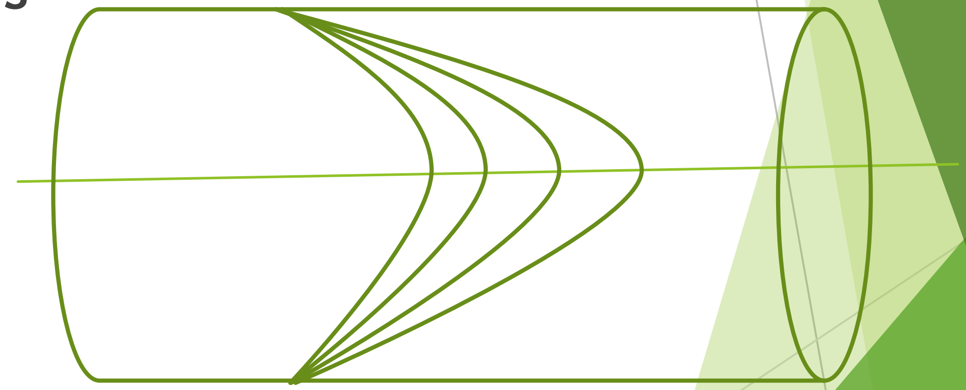
  Radius- r

  Pressure difference across 2 ends-P

Poiseuille's Equation

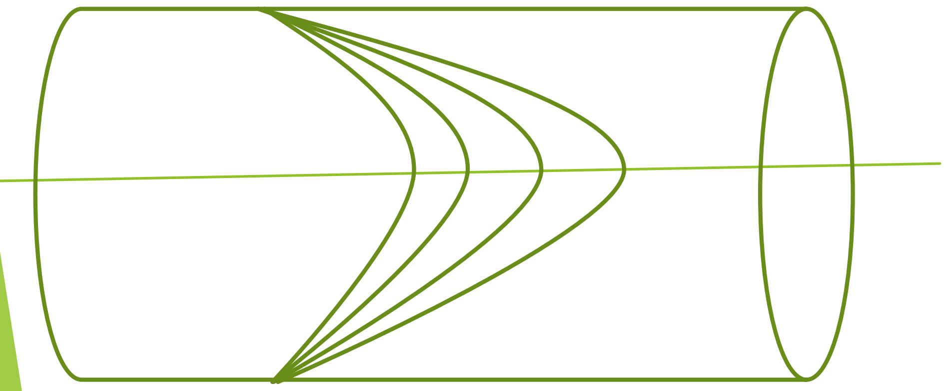
Rate of flow of a liquid through a tube

- Assumptions
  - ✓ The flow of liquid is steady and streamline
  - ✓ The tube is horizontal, so that gravity does not affect the flow of liquid
  - ✓ The liquid layer in contact with the tube remains stationary
  - ✓ The pressure is constant over any cross section, so that there is no radical flow of liquid.
  - ✓ The liquid yields only small shearing stress



Poiseuille's Equation

Rate of flow of a liquid through a tube



- Consider a cylindrical layer of

  x - radius

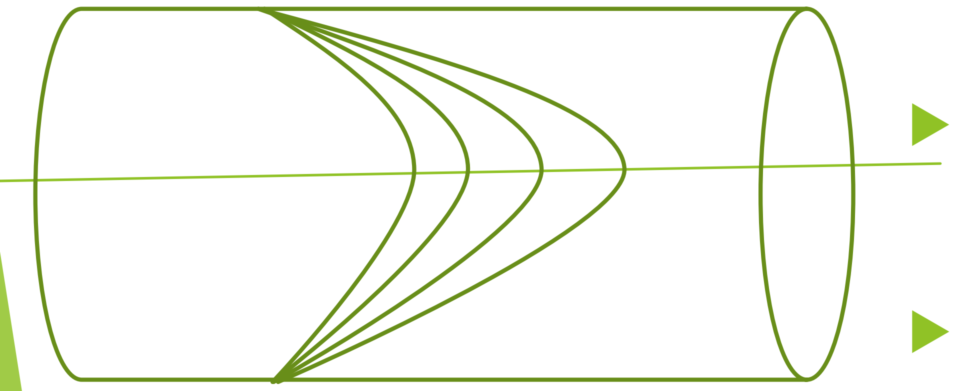
  v- velocity of all points in this layer

- Area of the layer A = 2πxl
- Viscous force acting on this layer

$$F = -\eta A \frac{dv}{dx} = -\eta 2\pi x l \frac{dv}{dx}$$

Poiseuille's Equation

Rate of flow of a liquid through a tube



- Due to thepressure difference between the ends of the tube, there is a forward force
- Force due to Pressure difference,

$$F = \pi x^2 P$$

- When the flow of the liquid is steady and streamline, these two forces are equal and opposite.

$$-\eta 2\pi x l \frac{dv}{dx} = \pi x^2 P$$

$$dv = -\frac{P}{2\eta l} x dx$$

Poiseuille's Equation

Rate of flow of a liquid through a tube

- Integrating, $v = -\frac{Px^2}{4\eta l} + C_1$
- To find C_1-Constant of Integration
  - when x=r, v=0 (The liquid layer in contact with the tube remains stationary)

$$0 = -\frac{Pr^2}{4\eta l} + C_1 \qquad C_1 = \frac{Pr^2}{4\eta l}$$

$$v = -\frac{Px^2}{4\eta l} + \frac{Pr^2}{4\eta l} = \frac{P}{4\eta l}(r^2 - x^2)$$

This is equation for a parabola. It shows that the velocity distribution curve is parabolic

Poiseuille's Equation

Rate of flow of a liquid through a tube

- Imagine a cylindrical shell Of radius x and thickness dx
- Cross sectional area of the shell, $dA = 2\pi x dx$
- Volume of liquid flowing through this area per second

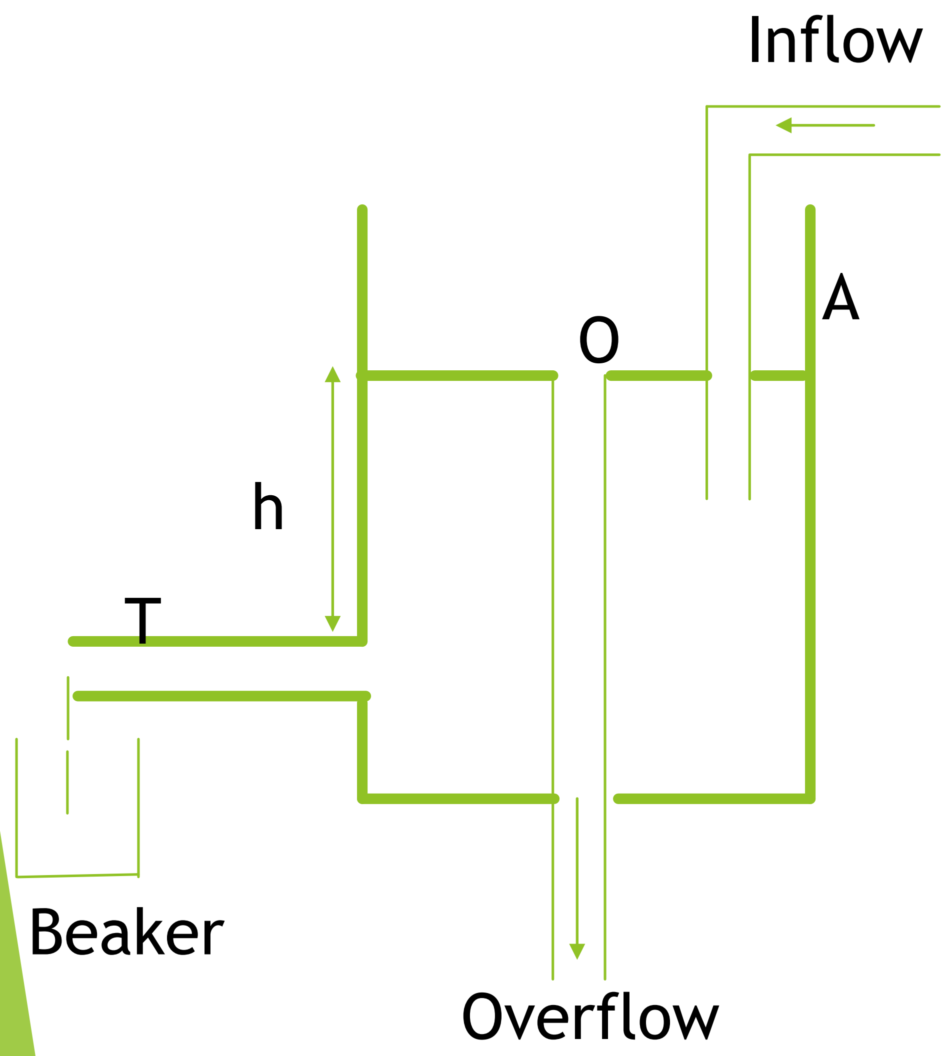
$$dV = vdA = \frac{P}{4\eta l}(r^2 - x^2)2\pi x dx = \frac{\pi P}{2\eta l}(r^2 - x^2)x dx$$

- Volume of liquid flowing through the tube per second= Integrating the expression within the limit x=0 to x=r

$$V = \frac{\pi P}{2\eta l}\int_0^r (r^2 - x^2)x dx = \frac{\pi P}{2\eta l}\int_0^r (r^2 x - x^3)dx = \frac{\pi P}{2\eta l}\left[\frac{r^2 x^2}{2} - \frac{x^4}{4}\right]_0^r$$

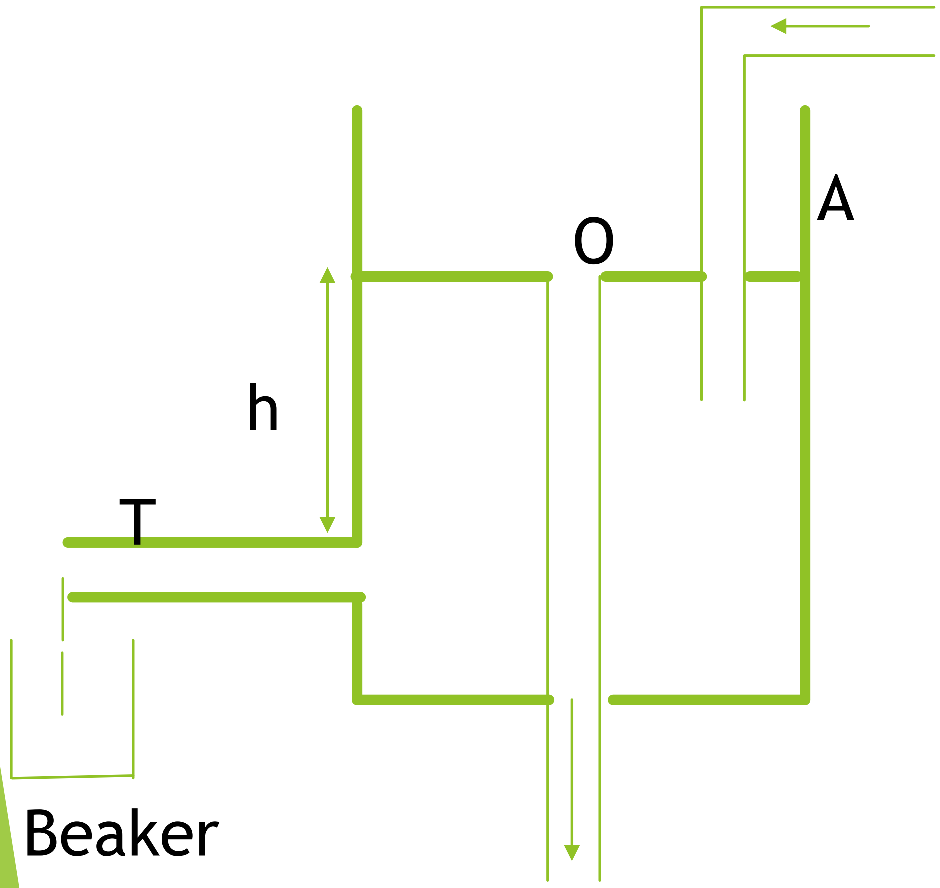
$$\boldsymbol{V = \frac{\pi P r^4}{8\eta l}}$$

Determination of coefficient of viscosity- Poiseuille's method



- Vessel A
- Constant Pressure head h-maintained by outflow arrangement
- Height of pressure head can be varied by adjusting O
- Capillary tube-T of length-l, radius-r fixed horizontally near the bottom of vessel
- A beaker of known weight is placed below the free end of the capillary tube

Determination of coefficient of viscosity- Poiseuille's method



- The weight of the dry beaker is taken
- The liquid flowing through the tube is collected for a known time
- The mass of the liquid collected is calculated
- Volume of the liquid $= \frac{Mass}{Density}$
- Rate of flow= $\frac{\text{Volume of the liquid}}{Time}$
- Coefficient of Viscosity,

$$\eta = \frac{\pi P r^4}{8Vl}$$

Derivation of Stoke's Equation

- Consider a spherical body fall through a viscous medium

- Assumptions
  - The spherical body is rigid and smooth
  - There is no slip between spherical body & medium'
  - The medium through which the body moves is of infinite extent
  - The medium is homogeneous
  - The diameter of the body is large compared with intermolecular distance of the medium
  - No waves or eddy currents set up in the medium during the motion of the body

Derivation of Stoke's Equation

- When a body falls through viscous medium, motion is opposed by viscous force

$$F = -\eta A \frac{dv}{dx}$$

- F increases with velocity
- When the viscous force=Gravitational force, Body attains constant velocity –Terminal Velocity
- According to Stoke's law,
- Viscous force, $F \propto v^a r^b \eta^c$

$$F = K v^a r^b \eta^c$$

F-Viscous Force
v-terminal velocity
r –radius
η -Coefficient of Viscosity
K-Dimensionless constant

$$F = Kv^a r^b \eta^c$$

- Taking dimensions

  $MLT^{-2} = (LT^{-1})^a L^b (ML^{-1}T^{-1})^c = M^c L^{(a+b-c)} T^{-(a+c)}$

- Comparing dimensions of M,

  C=1

- Comparing dimensions of T,

  -(a+c)=-2 a+1=2 a=1

- Comparing dimensions of L,
- (a+b-c)=1 b=1-a+c=1-1+1=1 **b=1**

$$F = Kvr\eta$$

$K=6\pi$

$$\boldsymbol{F = 6\pi v r \eta}$$

Derivation of Stoke's Equation

- When a body falls through a liquid,

$\rho - Density\ of\ the\ body$
$\sigma\ - D.ensity\ of\ the\ medium$

Weight of the body = $Mg = \frac{4}{3}\pi r^3 \rho g$ (Downward)

Upward thrust on the body by the medium = Weight of the liquid displaced

= $\frac{4}{3}\pi r^3 \sigma g$

Resultant downward force = $\frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g = \frac{4}{3}\pi r^3 (\rho - \sigma) g$

Derivation of Stoke's Equation

- When terminal velocity is attained,

$$\boldsymbol{6\pi\eta r v} = \frac{4}{3}\pi r^3(\rho - \sigma)g$$

$$\eta = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{v}$$

Terminal Velocity, $v = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

$\boldsymbol{v} \propto r^2$, $\boldsymbol{v} \propto (\rho - \sigma)$ $v \propto \frac{1}{\eta}$

Application of Stoke's law

$$6\pi\eta r v = \frac{4}{3}\pi r^3(\rho - \sigma)g$$

- To determine coefficient of viscosity of liquids
- To determine radius of small spherical objects like rain drops
- To determine electronic charge in Millikan's oil drop method

Application of Stoke's law Examples

- **Formation of cloud of tiny drops of water**
  - Tiny drops of water – small radius -0.001 cm
  - \- small Terminal velocity $\approx 1.2 cm/s$
  - They remain suspended in air
  - Appear to be floating

$\boldsymbol{v} \propto r^2$

Application of Stoke's law Examples

- **Rain drops**
  - Bigger drops of water – big radius -0.01 cm $v \propto r^2$

    - big Terminal velocity $\approx 120\ cm/s$
  - They fall through air

Application of Stoke's law Examples

- If $\rho > \sigma$, Terminal velocity = positive

The body will move downward

- If $\rho < \sigma$, Terminal velocity = negative

The body will move upward

Eg: Air bubbles formed in water

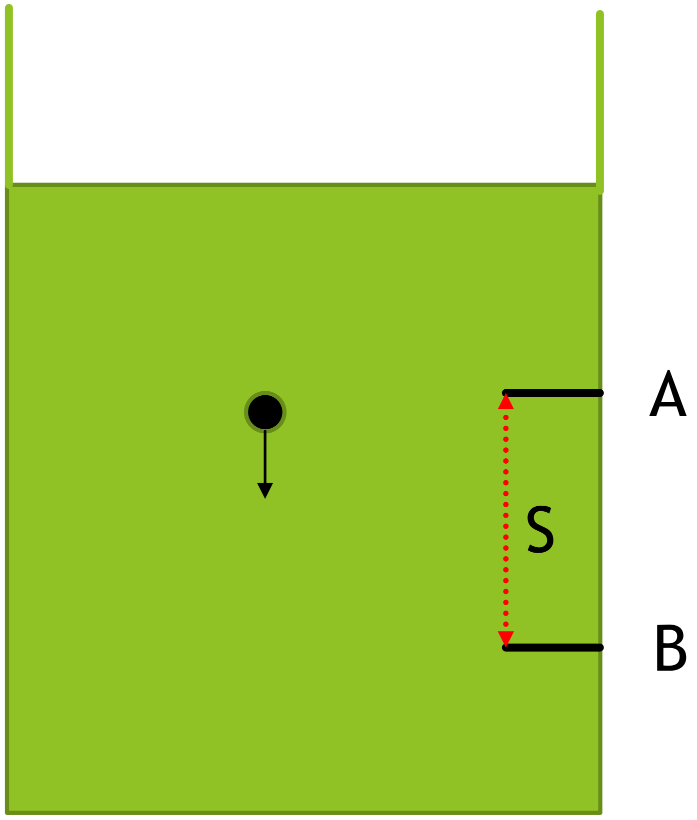
$$\boldsymbol{6\pi\eta r v} = \frac{4}{3}\pi r^3(\rho - \sigma)g$$

Terminal Velocity, $v = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

- For a small bubble, $\boldsymbol{v} \propto r^2$, terminal velocity small, small air bubbles will move up with small velocity

- When the size increases, $\boldsymbol{v} \propto r^2$, Terminal velocity increases

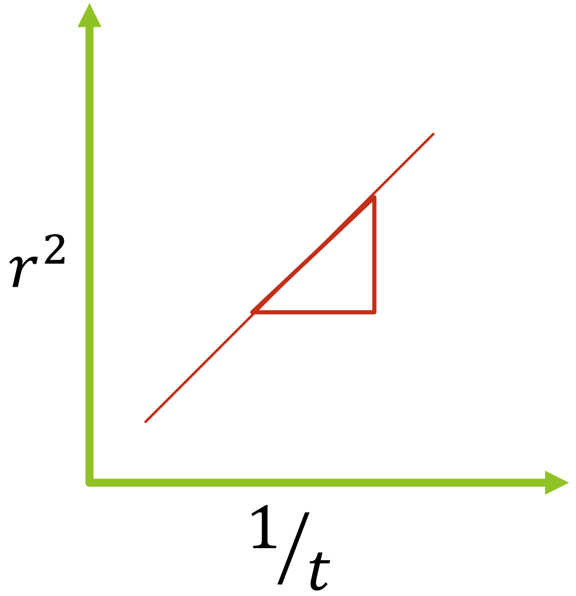
Determination of coefficient of viscosity- Stoke's falling viscometer



- The liquid whose η to be determined is taken in a jar
- Put two marks, A & B on the jar
- Tiny sphere of known radius is dropped centrally
- A stopwatch is started when the sphere just crosses A
- It is stopped just it cross B
- Distance AB=S, Time taken to cross AB=t
- Terminal Velocity, $v = \frac{S}{t}$

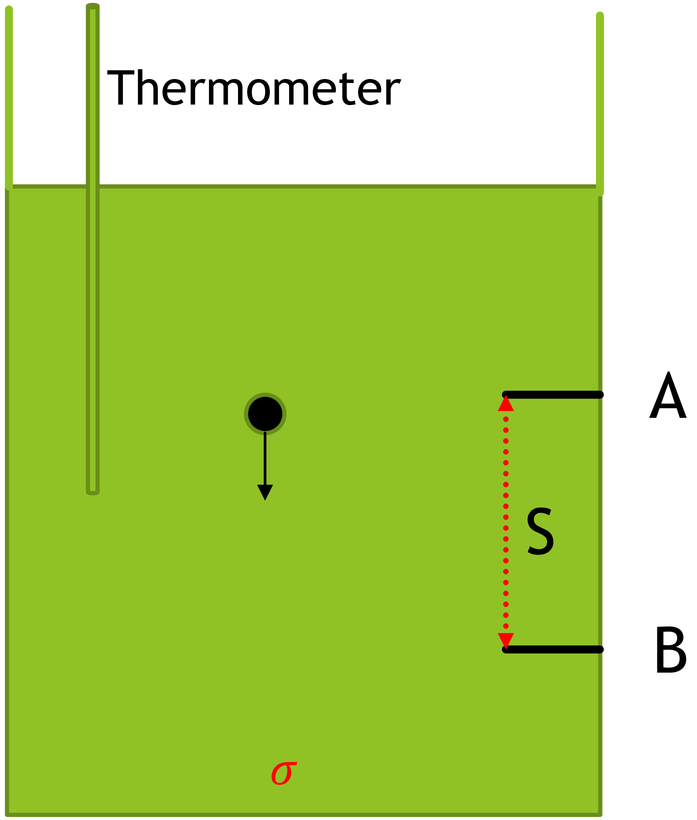
$$\eta = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{v} = \frac{2}{9}\frac{r^2 t(\rho - \sigma)g}{S}$$

Determination of coefficient of viscosity- Stoke's falling viscometer



- The experiment is repeated for spheres of different radii
- Time is noted in all cases
- A graph is plotted between r^2 and $^1/_t$
- It will be a straight line
- Slope=$\frac{dy}{dx} = \frac{r^2}{^1/_t} = r^2 t$
- r^2t is constant

Determination of coefficient of viscosity- Stoke's falling viscometer



- η for different temperatures can be found out
- A sensitive thermometer is used to measure the temperature of the liquid

Brownian Motion

- Brownian motion is the seemingly random movement of particles suspended in a fluid.
- It is the clearest proof of molecular agitation.
- It was first noticed by ROBERT BROWN in 1827
- Albert Einstein and Marian Smoluchowski predicted a solution for this
- Einstein's predictions were verified by Perrin and was awarded the Nobel prize for Physics

- According to Einstein,
  - The colloidal particle is struck by several molecules of dispersion medium
  - The movement is caused by unequal number of molecules of medium striking from opposite direction.
  - When more molecules strike the particle from one side than other direction of movement changes.
- Avg. Translational K.E= Avg K.E

$$\frac{1}{2} M\vec{V}^{2} = \frac{1}{2} m\vec{v}^{2} = \frac{3}{2} k_B T$$

M=Mass of colloidal particle
V=Velocity of colloidal particle
m=mass of molecules of medium at absolute temp. T
v=velocity of molecules of medium at absolute temp. T
k_B =Boltzmann's constant

- The Boltzmann's constant k_B is given by

$$k_B = \frac{R}{N_A}$$

R=Universal gas constant
N_A =Avogadro's number

- According to kinetic theory,

the mean Brownian displacement, $\bar{x}$ of a particle from its original position along a given axis after t seconds is,

$$\bar{x} = \left(\frac{RTt}{3\eta\pi r N_A} \right)^{\frac{1}{2}}$$

r=radius of particle
T=absolute temp.
η=coefficient of viscosity

Viscosity of gases

- Viscosity of gases arises from the molecular diffusion that transports momentum between layers of flow.
- The kinetic theory of gases allows accurate prediction of the behaviour of gaseous viscosity.
- Viscosity is independent of pressure.
- It increases as temperature increases.

Viscosity of liquids	Viscosity of gases
η_{Liquid} decreases with temperature	η_{gas} increases with temperature
While flowing, The molecules of liquid layer in contact with the walls remain stationary	Slipping occurs
Since the liquid is incompressible, $\frac{Volume}{t}$ is constant	Since density varies with pressure, $\frac{mass}{t}$ is constant

Meyer's formula

- Consider a gas flowing through a tube
- Let
    - V = volume of the gas flowing per second
    - X = distance from the inlet end of the tube
    - ρ= density of the gas
    - P = uniform pressure
- During the flow
    - density and volume of gas flowing through any section change
    - Mass of the gas flowing through any section taken to be constant

$$\rho V = constant \qquad \rho \propto P$$

$$PV = constant$$

- Consider
  - dx- A section of the tube
  - At a distance X from the inlet end
  - With a pressure difference dP

- Poiseuille's formula $V = \frac{\pi P r^4}{8\eta l}$
- Substituting for l=dx

  P=dP

$$V = \frac{\pi r^4}{8\eta}\frac{dP}{dx}$$

- As x increases P decreases,

$$V = -\frac{\pi r^4}{8\eta}\frac{dP}{dx}$$

- $PV = constant = K$

$$\mathbf{V = -\frac{\pi r^4}{8\eta}\frac{dP}{dx}}$$

- $PV = -\frac{\pi P r^4}{8\eta}\frac{dP}{dx} = K$
- $-\frac{\pi P r^4}{8\eta} dP = K dx$ $\quad\quad$ $-\frac{\pi r^4}{8\eta} PdP = K dx$
- $-\frac{\pi r^4}{8\eta}\int_{P_1}^{P_2} PdP = K\int_0^l dx$

P_1-Pressure at the inlet of the tube
P_2- Pressure at the outlet of the tube

- Integrating

$$-\frac{\pi r^4}{16\eta}({P_2}^2 - {P_1}^2) = Kl \quad\quad \frac{\pi r^4}{16\eta}({P_1}^2 - {P_2}^2) = Kl$$

$$K = \frac{\pi r^4}{16\eta l}({P_1}^2 - {P_2}^2)$$

- $PV = constant = K = P_1V_1 = P_2V_2$

$$P_1V_1 = P_2V_2 = \frac{\pi r^4}{16\eta l}({P_1}^2 - {P_2}^2)$$

This is Meyer's formula for gaseous flow through a capillary tube

Effect of pressure on the viscosity of gases

- James Clerk Maxwell published a paper in 1866 explained gaseous viscosity using the kinetic theory of gases
- The viscosity coefficient ∞ density (pressure),

  ∞ mean free path

  ∞ mean velocity of atoms
- mean free path ∞ $\frac{1}{\text{density (pressure)}}$
- So increase of pressure doesn't change viscosity
- But at high pressures, Viscosity of gases increases with pressure

Effect of temperature on the viscosity of gases

- The viscosity of gases increases with temperature
- Sutherland's formula can be used to derive the viscosity of an ideal gas as a function of the temperature

$$\eta = \eta_0 \left(\frac{T_0 + C}{T + C}\right)\left(\frac{T}{T_0}\right)^{3/2}$$

η = Viscosity in (Pa.s) at input temperature T
η_0=Reference Viscosity in (Pa.s) at reference temperature T_0 (273 K)
C = Sutherland's constant for the gas

- This equation is valid for temperatures between 0 < T < 555 K

Thank You